CH – 6 SIMILAR TRIANGLES

1. Similar Figures :

Two geometrical figures are said to be similar figures, if they have same shape but not necessarily the same size.

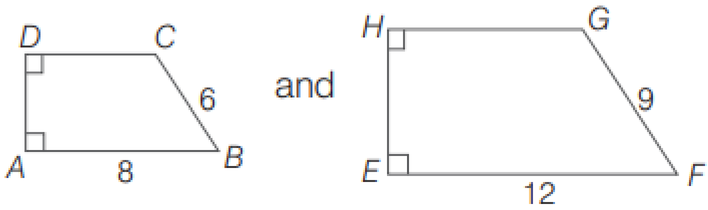
Or

A shape is said to be similar to other, if the ratio of their corresponding sides is equal and the corresponding angles are equal.

Note Principle of similarity is used in measuring the heights and distance of objects like mountain or moon.

2. Similar Polygons :

Two polygons of the same number of sides are similar, if
(i) all the corresponding angles are equal and
(ii) all the corresponding sides are in the same ratio (or proportion).



If only one condition from (i) and (ii) is true for two polygons, then they cannot be similar.

3. The ratio that compares the measurements of two similar shapes, is called the **scale factor** or **representative fraction**. It is equal to the ratio of corresponding sides of two figures. We can use the ratio of corresponding sides to find unknown sides of similar shapes.

4. Two triangles are said to be **similar triangles**, if their corresponding angles are equal and their corresponding sides are proportional (i.e., the ratios between the lengths of corresponding sides are equal).

e.g., If in ΔABC and ΔPQR,
∠A = ∠P, ∠B = ∠Q, ∠C = ∠ R

and $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

then , ΔABC ~ ΔPQR

where, symbol ~ is read as, 'is similar to'.

Conversely
If ΔABC is similar to ΔPQR, then
∠ A = ∠ P, ∠ B = ∠Q, ∠ C = ∠ R

and $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

Note The ratio of any two corresponding sides in two equiangular triangles is always the same.

5. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then other two sides are divided in the same ratio. This theorem is known as **Basic Proportionality Theorem (BPT) or Thales theorem.**

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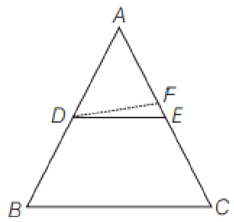
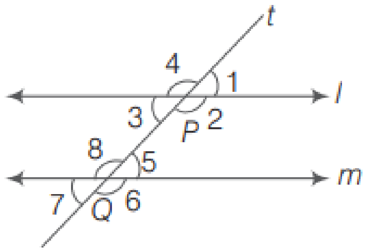
6. If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side (converse of basic proportionality theorem).



7. Let us consider two parallel lines l and m and draw one transversal line t, which intersect l and m at P and Q. Then,



(i) Corresponding Angles

The angles on the same side of a transversal line are known as the corresponding angles, if both lie either above the two lines or below the two lines. i.e.,

(a) ∠1and ∠5 (b) ∠2 and ∠6
(c) ∠4 and ∠8 (d) ∠3 and ∠7

(ii) Alternate Interior Angles

The following pairs of angles are called pairs of alternate interior angles

(a) ∠3 and ∠5 (b) ∠2 and ∠8

(iii) Consecutive Interior Angles

The pairs of interior angles on same side of the transversal line are called pair of consecutive interior angles.

(a) ∠2 and ∠5 (b) ∠3 and ∠8

(iv) Alternate Exterior Angles

The following pair of angles are called alternate exterior angles

(a) ∠1 and ∠7 (b) ∠4 and∠6

(v) Vertically Opposite Angles

The following pair of angles are called vertically opposite angles.

(a) ∠1and ∠3 (b) ∠2 and ∠4
(c) ∠5 and ∠7 (d) ∠6 and ∠8

8. The line segment joining the mid-points of any two sides of a triangle is parallel to the third side. (Mid-point theorem)

9. The internal bisector

of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle. (Angle bisector theorem)

10. Similar Triangles

Two triangles are said to be similar, if corresponding angles are equal and corresponding sides are in the same ratio i.e., they are proportional.

11. AAA Similarity

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar (because by the angle sum property of a triangle, their third angle will also be equal) and it is called **AAA (or AA) similarity**.

12. SSS Similarity

In two triangles, if sides of one triangle are proportional to the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.

13. SAS Similarity

If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

14. The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

If ΔABC and ΔPQR are similar, then

$$\frac{ar(\Delta \mathrm{ABC})}{ar(\Delta \mathrm{PQR})} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{AC}{PR}\right)^2 = \left(\frac{BC}{QR}\right)^2$$

15. (i) In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. (Pythagoras theorem)
(ii) In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to first side is a right angle. (Converse of Pythagoras theorem)

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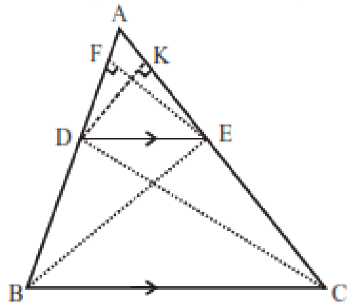
TH.1 : In a triangle, a line drawn parallel to one side to intersect the other two side in distinct points divides the two sides in the same ratio.

Given : A ΔABC in which DE is drawn parallel to side BC. Intersects AB and AC at point D and respectively.



To Prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Const.: Join BE and CD and drawn $EF \perp AB$ and $DK \perp AC$.

Proof :

$$ar(\Delta ADE) = \frac{1}{2} \times AD \times EF \qquad (i)$$

$$ar(\Delta BDE) = \frac{1}{2} \times BD \times EF \qquad (ii)$$

Equation (i) divided by (ii)

$$\Rightarrow \frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2} AD \times EF}{\frac{1}{2} BD \times EF}$$

$$\Rightarrow \frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{AD}{BD} \qquad (iii)$$

$$\text{Again } ar(\Delta ADE) = \frac{1}{2} \times AE \times DK \qquad (iv)$$

$$ar(\Delta CDE) = \frac{1}{2} EC \times DK \qquad (v)$$

Equation (iv) divided by (v)

$$\Rightarrow \frac{ar(\Delta ADE)}{ar(\Delta CDE)} = \frac{\frac{1}{2} AE \times DK}{\frac{1}{2} EC \times DK}$$

$$\Rightarrow \frac{ar(\Delta ADE)}{ar(\Delta CDE)} = \frac{AE}{EC} \qquad (vi)$$

Since, Δ's BDE and CDE lies on same base DE and same parallel BC.

$$\therefore \quad ar(\Delta BDE) = ar(\Delta CDE)$$

From (iii) and (vi)

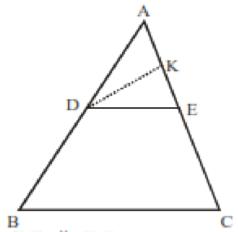
$$\frac{AD}{BD} = \frac{AE}{EC}$$ **Hence Proved**

COROLLARY

$\frac{AD}{DB} = \frac{AE}{EC}$	(i)	$\frac{DB}{AD} = \frac{EC}{AE}$	(iv)
$\frac{AB}{DB} = \frac{AC}{EC}$	(ii)	$\frac{AB}{AD} = \frac{AC}{AE}$	(v)
$\frac{DB}{AB} = \frac{EC}{AC}$	(iii)	$\frac{AD}{AB} = \frac{AE}{AC}$	(vi)

Th.2 : Convers of B.P.T. : If a line divides any two sides of a triangle in the same ratio, then line is parallel to the third side.

Given : A ΔABC, DE is a line divides two sides AB and AC in the same ratio that is $\frac{AD}{DB} = \frac{AE}{EC}$



To prove : DE $\parallel$ BC

Const.: If DE is not $\parallel$BC, draw DK $\parallel$ BC meeting AC in K.

Proof : A ΔABC, DK $\parallel$ BC (By const.)

$$\therefore \quad \frac{AD}{DB} = \frac{AK}{KC} \quad \text{(By B.P.T.)} \qquad (i)$$

$$\text{But } \frac{AD}{DB} = \frac{AE}{EC} \quad \text{Given} \qquad (ii)$$

From (i) and (ii)

$$\frac{AK}{KC} = \frac{AE}{EC}$$

This is possible only one condition when point K conside with E. Therefore DE $\parallel$ BC.

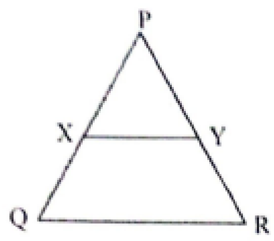
if there :

(a) Corresponding angules are equal.

(b) Corresponding sides are proportional.

Practice sheet 1

Q1.In the following figure , XY II QR and $\frac{PX}{XQ} = \frac{PY}{YR} = \frac{1}{2}$, then



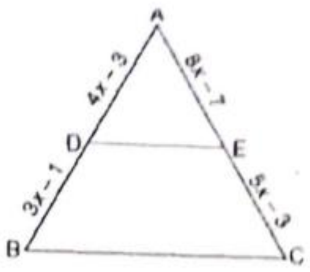
(a) XY = QR

(b) XY = $\frac{1}{3}$ QR

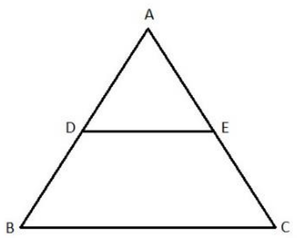
(c) $XY^2 = QR^2$

(d) XY = $\frac{1}{2}$ QR

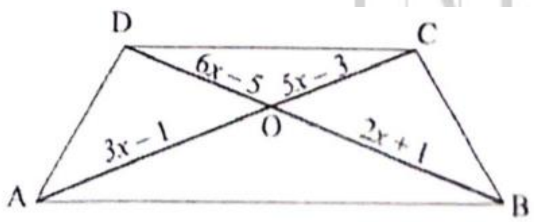
Q2. In the figure DE II BC .Find x .



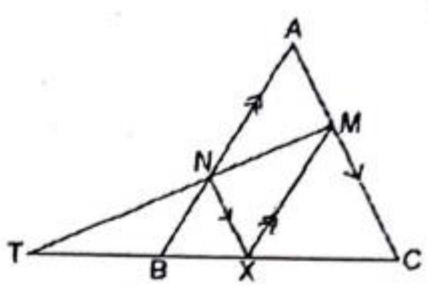
Q3. In the given Fig. 1 DE is parallel to BC.If $\frac{AD}{DB} = \frac{2}{3}$ and AC = 18 cm ,Find AE .



Q4..In the given figure , AB II DC and diagonals AC and BD intersects at O .If OA = 3X – 1 and OB = 2x + 1 , OC = 5x – 3 and OD = 6x – 5 , find the value of x .

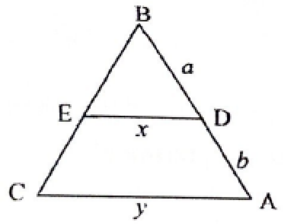


Q5.In the figure , XN II CA and XM II BA .T is a mid point on CB produced .Prove that TX^2 = TB .TC .

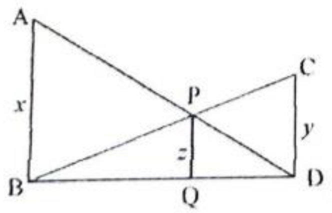


Q6. In the given figure DE II AC . Which of the following is correct ?

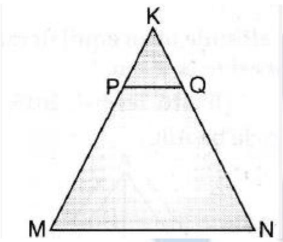
X = $\frac{a+b}{ay}$ or x = $\frac{ay}{a+b}$



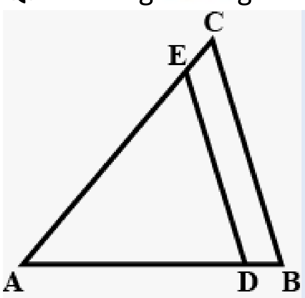
Q7.In the given figure AB II PQ II CD , AB = x , CD = y and PQ = z . Prove that $\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$.



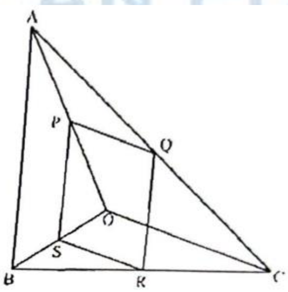
Q8. In the given Fig. 2 PQ is parallel to MN. If $\frac{KP}{PM}=\frac{4}{13}$ and KN = 20 cm ,find KQ.



Q9. In the given Fig.3. DE ||BC. If AD = x , DB = x - 2 , AE = x + 2 and EC= x - 1, find the value of x.



Q10. In the figure ,If PQRS is a parallelogram and AB II PS ,then prove that OC II SR .



Q11. Any point X is taken in the side BC of a triangle ABC, and XM, XN are drawn parallel to BA, CA meeting CA, BA in M, N respectively; MN meets BC produced in T, prove that TX^2 = TB x TC.

Q12. Two triangles ABC and DBC lie on the same side of the base BC, From a point P on BC, PQ||AB and PR||BD are drawn. They met AC in Q and DC in R respectively. Prove that QR||AD.

Q13. Let ABC be a triangle and D and E be two points on side AB such that AD=BE. If DP||BC and EQ||AC, then prove that PQ||AB.

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Q14. In ΔABC, D and E are points on the sides AB and AC respectively. For each of the following cases show that DE||BC.

(a) AB=12cm, AD=8cm, AE=12cm and AC=18cm.

(b) AB=5.6cm, AD=1.4cm, AC=7.2cm and AE=1.8cm.

Q15. In a ΔABC, D and E are points on the sides AB and AC respectively such that DE||BC. If AD=6cm, DB=9cm and AE=8cm. Find AC.

Q16. In a ΔABC, D and E are points on the sides AB and AC respectively such that DE||BC. If $\frac{AD}{DB} = \frac{3}{2}$ and AC=15cm, find AE.

Q17. In a ΔABC,D and E are points on the sides AB and AC respectively such that DE||BC.If AE = 8 , DB = X – 1 and EC = 3 and AD = x , find x.

Q18.In a Δ ABC , D and E are points on the sides AB and AC respectively such that DE II BC .If $\frac{AD}{BD} = \frac{4}{5}$ and EC = 2.5 cm , find AE .

Q19. In a ΔABC , D and E are points on sides AB and AC respectively such that DE||BC. If AD= x, = DB = x - 2, AE = x + 2and EC = x – 1 , find the value of x.

Q20. In a ΔABC, D and F are points on the sides AB and AC respectively such that DE||BC. If AD = 4x - 3, AE= 8x - 7 , BD = 3x - 1 and CE = 5x - 3, find the volume of x.

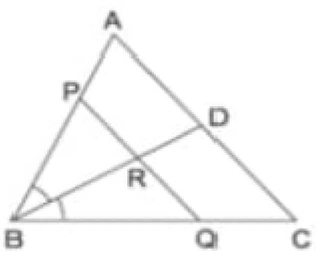
Q21. In Δ ABC, P and Q are points on sides AB and AC respectively, such that PQ||BC. If AP = 3.4cm, AQ = 2cm, QC = 3cm and BC = 6cm, find AB and PQ.

Q22. In ΔABC, DE || BC. It AD 3, DB = 4, AC = 12cm . find the value of AE.

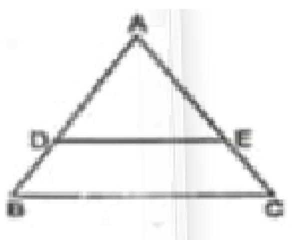
Q23. In ABC, DE || BC if $\frac{AD}{DB} = \frac{a}{b}$ and AC = 2a find the value of AE.

Q24. In ΔPQR, MN || QR it PQ = a+ b , MQ = a and PR = 2b find NR.

18. From the below figure , In ΔABC , $the\ bisector\ of\ \angle B\ meets\ AC\ at\ D$ .A line PQ II AC meets AB , BC and BD at P ,Q and R respectively .Show that PR X BQ = QR X BP.



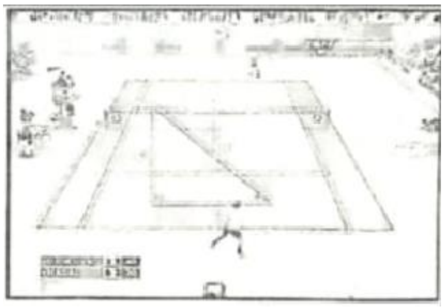
Q25.If a line is drawn parallel to one side of a triangle intersecting the other sides in distinct points ,then prove that the other two sides are divided in the same ratio. Using the above .In this fig. DE II BC , AD = $\frac{1}{2}\ BD\ ; AE = 4.5\ cm$, find the value of AC.

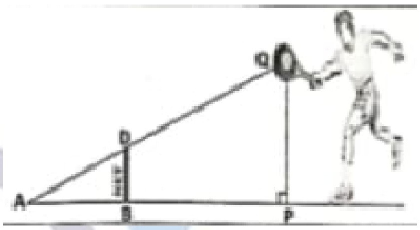


Q26.(i) Prove that if a line is drawn to one side of a triangle to intersect the other two sides in distinct points ,the other two sides are divided in the same ratio.

(ii) In ΔABC , D and E are points on the side AB and AC respectively such that DE II BC .If $\frac{AD}{DE} = \frac{3}{4}$ and AC = 15 cm find AE .

Q27. Tennis is an individual sport at first glance as the player is obviously alone on the court betting it out with his opponent .How does knowledge of mathematics help us gain insight into the world of tennis ?



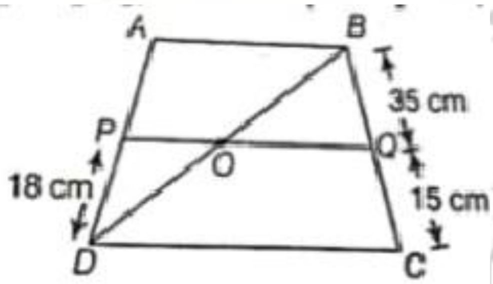


(a) In the given figure BD II PQ , find the value of x .

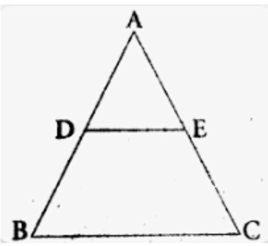
AB= x- 2 , BP = X – 1 ,
AD = x + 3 & DQ = x + 5

(b) If one of the diagonal of a trapezium divides the other in the ratio 2 : 1 .Prove that one of the parallel sides is twice the other .

(c) In the figure AB II CD II PQ , find AD (with proof)



Q28. State and proof BPT .Hence in this fig. DE II BC and BD = x – 3 , AB = 2x , CE = x-2 and AC = 2x + 3 , find x.



Q29.ABCD is a parallelogram .AB is divided at P and CD at Q so that AP : PB = 3 : 2 and CQ : QD = 4 : 1 .If one student is less in each row, there would be 3 rows more .Find the total number of students in the class.

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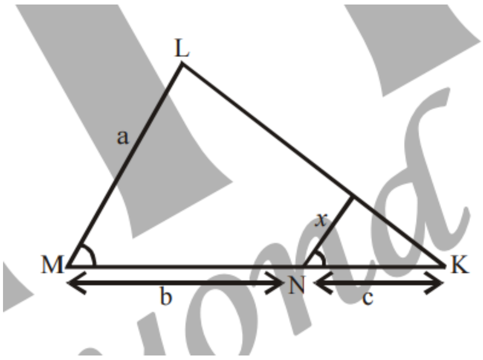
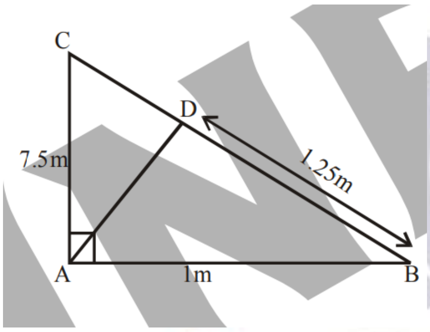
Practice Sheet – 2

Q30. In the given Fig. express x in terms of a, b and C.



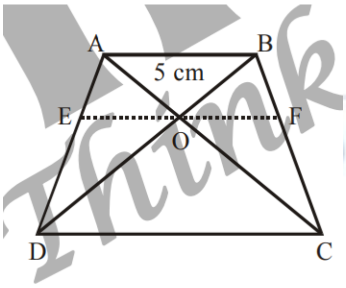
Q31.The perimeter of two similar ∆s ABC and PQR are respectively 36cm and 24cm. If PQ = 10cm, find AB.

Q32. In the given Fig. 10. ∠ CAB = 90^0 and AD ⊥ BC, If AC = 7.5m, AB = 1m and BD=1.25 m find AD.

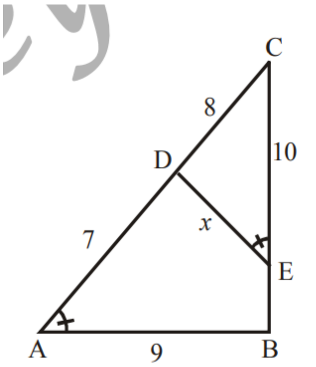


Q33. P and Q are points on sides AB and AC respectively of ∆ABC. If AP = 3cm, PB = 6cm, AQ = 5cm and QC = 10cm, show that BC = 3PQ.

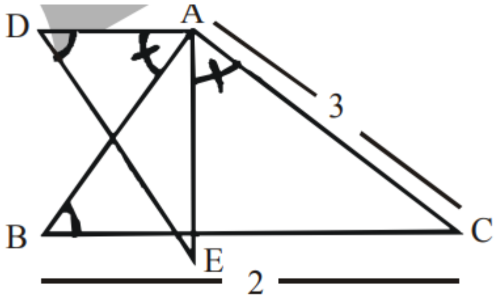
Q34. In the given Fig. 11 ABCD is a trapezium and DC = 2 AB. EF is drawn parallel to AB such that BE : EC = 3 : 4 , prove that 7EF = 10AB .



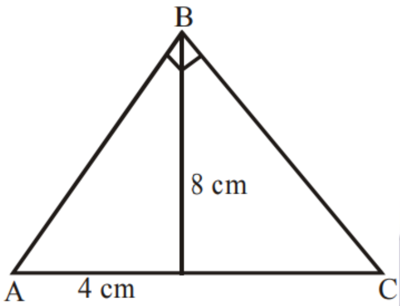
Q35. In the given fig. if $\angle A = CED$, proved that ∆CAB ~ ∆ CED .Also find the value of x .



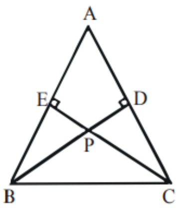
Q36. In the given fig.13 $\angle BAD = \angle CAE$ and $\angle ADE = \angle ABC$ .If AC : BC = 3 : 2 , find the ratio DE : AE .



Q37. In the given fig. 15 $\angle$ ABC = 90^0 and BD$\perp$ AC. If BD = 8cm and AD = 4cm, find CD.



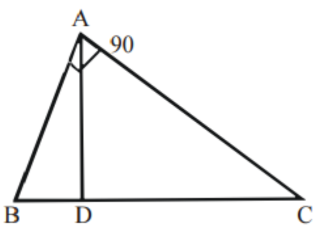
Q38. In the given fig. 16 considering triangles BE P and CPD , prove that BP X PD= EP X PC.



Q39. ABC is a triangle in which AB = AC and D is a point on the side AC such that BC^2 = AC X CD. Prove that BD = BC

Q40. Two sides and the a median bisecting one of these sides of a Δ are respectively proportional the two sides and the corresponding median of the other triangle. Prove that Δ's are similar.

Q41. In the given Fig. 17, $\angle$BAC = 90^0 and segment AD $\perp$ BC. Pt. AD^2 = BD X DC.



Q42. In the given Fig. 18 ABCD is a trapezium such that AB||DC. Diagonals AC and BD interest each other to, Prove that $\frac{OA}{OC} = \frac{OB}{OD}$.

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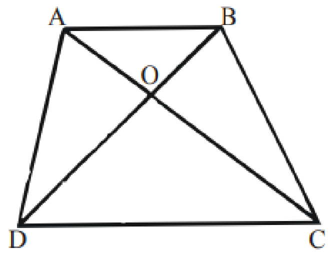
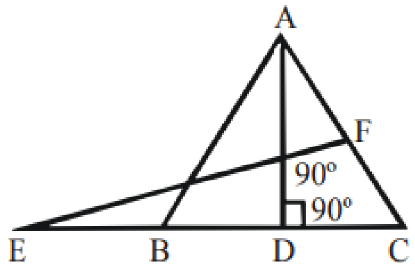


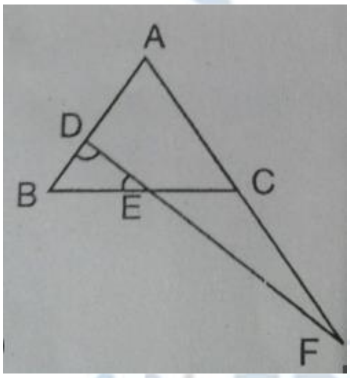
Q43.In the given fig. 19 $\angle ABC = 90^0$ and BD $\perp$ AC if AB = 5.7cm ,BD = 3.9 cm and CD = 5.4 cm and find BC.



Q44.The diagonal BD of a parallelogram ABCD intersects the segment AE at the point F, where E is any point on the side BC.Prove that DF X EF = FB X FA.

Q45.Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM od another triangle PQR .Show that $\Delta ABC \sim \Delta\, PQR$.

Q46. In the fig, $\angle BED = \angle BDE$ and E id the midpoint of BC.Prove that AF X BE = AD X CF .



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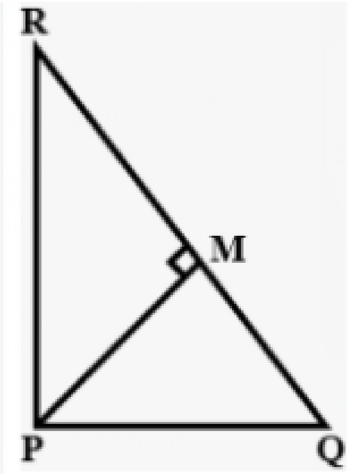
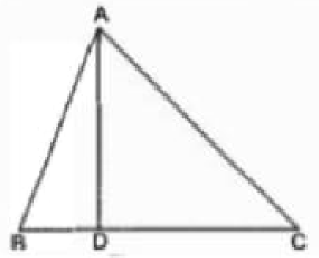
Q47.PQR is a right angled at P and M is a point on QR such that PM $\perp$ QR .Show that PQ^2 = QM X QR .



Q48. In the fig. $\angle BAC = 90^0$ and segment AD $\perp BC$ .Prove that AD^2 = BD X DC .



Q49.In a ΔABC , $AD \perp BC$ and AD^2 = BD X CD .Prove that ΔABC is a right triangle.

Q50. If sides AB , BC and median AD of ΔABC are prportional to the corresponding sides PQ , QR and median PM of PQR , Show that $\Delta ABC \sim \Delta PQR$.

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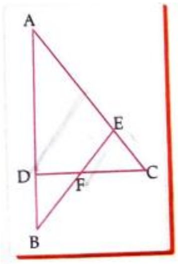
ADDITIONAL QUESTIONS

Q51.(a)State and prove BPT.
(b) In the given fig.$\angle CEF = \angle CFE$.F is the midpoint of DC.
Prove that $\frac{AB}{BD} = \frac{AE}{FD}$.

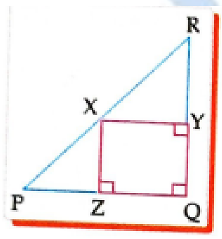


Q52.State and Prove BPT.In ΔABC , $if\ DE\ II\ BC$,AD = x ,DB = x – 2 , AE = x + 2 and EC = x– 1 ,then using the above result find the value of x .

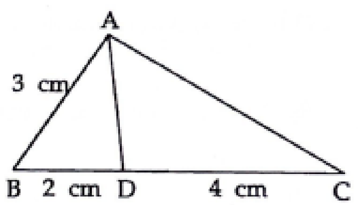
Q53.D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$, prove that CA^2 = CB.CD.

Q54.In ΔPQR , N is a point on PR , such that QN $\perp PR$.If PN x NR = QN^2.Prove that $\angle PQR = 90^0$.

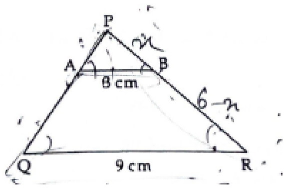
Q55. ΔPQR is a right angled at Q.QX $\perp$ PR , XY $\perp RQ$ and XZ $\perp PQ$ are drawn.Prove that XZ^2 = PZ X ZQ .



Q56.In the given figure,AD is the bisector of $\angle A$ .Find AC.

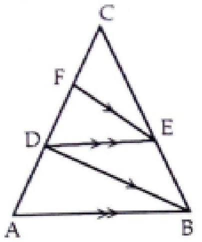


Q57. In the given figure ,PR = 6cm and AB II QR .Find BP.

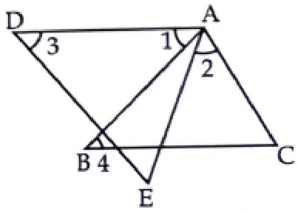


Q58.ABCD is a trapezium with AD II BC and AD = 4cm.If the digonals AC and BD intersect each other at O such that $\frac{AO}{OC} = \frac{DO}{OB} = \frac{1}{2}$, then find BC.

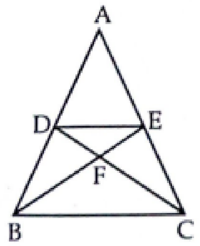
Q59.In the given fig, AB II DE and BD II EF .Prove that DC^2 = CF X AC .



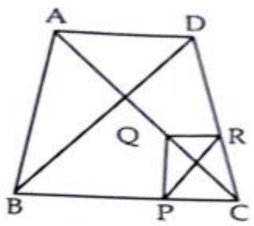
Q60.In the given fig.$\angle 1 = \angle 2$ and $\angle 3 = \angle 4$.Show that AE.BC = AC.DE.



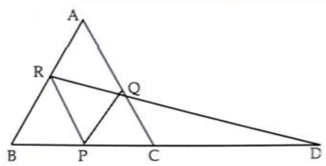
Q61.In the given fig.$\Delta ABE \cong \Delta ACD$, Prove that $\Delta ADE \sim \Delta ABC$.



Q62.In the given fig,two triangles ABC and DBC lie on the same side BC such that PQ II BA and PR II BD.Prove that QR II AD.



Q63.In the given fig. PQ II BA ;PR II CA .If PD = 12 cm .Find BD X CD.

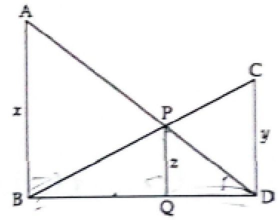


Q64.In ΔABC ,P divides the side AB such that AP : PB = 1: 2 .Q is a point on AC such that PQ II BC.Find the ratio of the areas of ΔAPQ and trapezium BPQC.

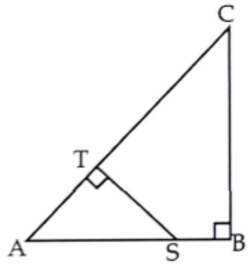
Q65. Through the mid point M of the side CD of a parallelogram ABCD ,the line BM is drawn intersecting AC in L and AD produced in E.Prove that EL = 2BL.

Q66.Two poles of height p and q metres are standing vertically on a level ground 'a' metres apart.Prove that the height of the point of intersection of the lines joining the top of ech pole to the foot of the opposite pole is given by $\frac{pq}{p+q}$ metres.

Q67.In fig.AB IIPQIICD ,AB = x metres ,CD = y units and PQ = z units , prove that $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$.



Q68.In the given fig.$\angle T$ and $\angle B$ are right angles.If the lengths of AT ,BC and AS (in cm) are 15,16 and 17 respectively ,then find the length of TC (in cm).



Q69.In ΔABC is such that AB = 3cm ,BC = 2cm and CA = 2.5 cm.If $\Delta DEF \sim \Delta ABC$ and EF = 4cm , then find perimeter of ΔDEF .

Q70.Sides of two similar triangles are in the ratio of 4 : 9 .Find the ratio of the areas of the triangles.

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