

CH – 5 ARITHMETIC PROGRESSION

Sequence :

A sequence is an arrangement of numbers in a definite pattern according to some rule

Arithmetic Progression :

A sequence in which each term differs from its preceding term by a constant is called an arithmetic progression, written as an A.P. The constant difference is called (Common difference).

To find the n^{th} Term of an A.P

Theorem : If the first term of an AP is 'a' and its common difference is d then show that nth term is given by $a_n = a(n - 1) \times d$.

Proof :

In the given AP, we have, First term = a, and common difference = d So, the Given AP may be written as a, a + d, a + 2d, a + 3d, a + 4d, a + 5d, in this AP, we have

$$\text{First term, } a_1 = a = a + (1 - 1)d$$

$$\text{Second term} = a_2 = a + d = a + (2 - 1)d$$

$$\text{Third term} = a_3 = a + 2d = a + (3 - 1)d$$

$$\therefore \text{nth term } (a_n) = a + (n - 1)d$$

Note : The nth term of an AP is called General Term. To find the nth term from the end of an AP.

Theorem :

If a be the first term, d the common difference and l is the last term of a given AP then show that its nth term from the end is $\{l - (n - 1)d\}$

Proof : We may write the given AP as

$$a, (a + d), (a + 2d), \dots, (l - d), l$$

Thus, we have

$$\text{Last term} = l = l - (1 - 1)d$$

$$\text{2nd term from the end} = (l - d) = \{l - (2 - 1)d\};$$

$$\text{3rd term from the end} = (l - 2d) = \{l - (4 - 1)d\};$$

$$\therefore \text{nth term from the end} = \{l - (n - 1)d\}$$

Important Results :

It is always convenient to make a choice of

(i) 3 No's in AP as $(a - d), a, (a + d)$

(ii) 4 No's in AP as $(a - 3d), (a - d), (a + d), (a + 3d)$

(iii) 5 No's in AP as $(a - 2d), (a - d), a, (a + d), (a + 2d)$

Derivation of sum Formula

1. If an A.P has first term 'a' and common difference 'd', then the sum of the first n terms is given by :

$$S_n = \frac{n}{2} [2a + (n - 1)d] \text{ or } S_n = \frac{n}{2} (a + a_n)$$

where l is the last term of A.P, $l = a_n$.

CLASS X

Proof :

Let a_1, a_2, a_3, \dots be an A.P. with first term 'a' and common difference 'd'. Then,

$$a_1 = a ; a_2 = a + d ; a_3 = a + 2d \quad a_n = a + (n - 1) d$$

$$S_n = a_1 + a_2 + \dots + a_{n-1} + a_n$$

$$S_n = a + (a + d) + \dots [a + (n - 2)d] + [a + (n - 1) d] \quad (i)$$

Writing the above series in reverse order

$$S_n = \{a + (n - 1)d\} + \{a + (n - 2)d\} + \dots + (a + d) + a \quad (ii)$$

Adding (i) and (ii), we get

$$2S_n = [2a + (n - 1)d] + \dots [2a + (n - 1) \times d]$$

$$2S_n = n [2a + (n - 1) d]$$

$\therefore 2a + (n - 1) \times d$ repeated n times.

$$S_n = \frac{n}{2} [2a + (n - 1) \times d] \quad \text{OR} \quad S_n = \frac{n}{2} (a + a_n)$$

Where, a_n = last term or nth term.

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Practice sheet – 1

- Q1.** Write the first three terms of the sequence defined by $a_n = 3n + 2$.
- Q2.** Write the first five terms of the sequence is defined by $a_n = \frac{n-2}{3}$.
- Q3.** Write the first three terms of the sequence is defined by $a_n = \frac{3n-2}{5}$.
- Q4.** Find the missing terms of the AP 3, ____, 33, 48.
- Q5.** For an AP, if $a_{25} - a_{20} = 35$, then find common difference.
- Q6.** Find the next term of the A.P $\sqrt{3}, \sqrt{12}, \sqrt{27}, \dots$
- Q7.** Show that sequence 71, 70, 69, 68..... is an A.P.. Find its 13th term and the general term.
- Q8.** Which term of the A.P. 21, 42, 63, 84, 105, 126 is 420.
- Q9.** Suppose that an arithmetic sequence has $a_8 = -16$ and $a_{16} = -40$ find a.
- Q10.** If the nth term of an A.P. is $(5n - 2)$, find (i) first term (ii) common difference (iii) 19th term.
- Q11.** The 10th term of an A.P. is 41 and the 18th term is 73 ; find the Arithmetic progression.
- Q12.** If $(K + 1)$, $3K$ and $(4K + 2)$ be any three consecutive terms of an A.P., find the value of K.
- Q13.** Is 310 a term of the A.P. 3, 8, 13, 18,
- Q14.** For what value of n, the nth term of the series "9, 7, 5....." and "15, 12, 9," is the same.
- Q15.** Find the three numbers in A.P., Whose sum is 21 and whose product is 315.
- Q16.** Find the four numbers in A.P whose, sum is 50 and in which the greatest number is 4 times the least.
- Q17.** The sum of three consecutive terms of an A.P. is 15 and the sum of their squares is 83. Find the terms.
- Q18.** The sum of five numbers in A.P. is 25 and the sum of their squares is 165. Find the numbers.
- Q19.** The sides of a right angle-triangle are in A.P. Show that they are in the ratio 3 : 4 : 5.
- Q20.** Divide 32 into four parts which are in A.P. such that the product of extremes is to the product of means is 7 : 15.
- Q21.** Find the all three digit numbers are divisible by 7.
- Q22.** The 16th term of an AP is 1 more than twice its 8th term, if the 12th term of an AP is 47, then find its nth term.
- Q23.** Find the 8th term from the end of the AP 7, 10, 13, ... 184.
- Q24.** The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.
- Q25.** Two AP's have the same common difference. The difference between their 100th term is 111222333. What is the difference between their millionth terms.

Practice sheet – 2

- Q27.** Find the sum of first 19 terms of the AP 2, 7, 12, 17,...
- Q28.** Find the sum of first 18 terms of the AP 9, 7, 5, 3, ...
- Q29.** Which term of the AP 5, 9, 13, 17, is 81 ? Also find the sum (5 + 9 + 13 + ... + 81).
- Q30.** Find the sum of first 20 terms of the given AP $1/2, 1/3, 1/6 + \dots$
- Q31.** Find the sum of all three-digit natural numbers which are divisible by 13.
- Q32.** Find the sum of first hundred even natural numbers which are divisible by 5.
- Q33.** If the 5th and 12 terms of an AP are -4 and -18 respectively, find the sum of the first 20 terms of the AP.
- Q34.** The sum of first n terms of an AP is given by $(n^2 + 8n)$. Find the 12th term of the AP. Also find the n th term of the AP
- Q35.** If the sum of the first n terms of an AP is given by $S_n = (3n^2 - n)$, find its (i) n th term (ii) first term, and (iii) common difference.
- Q36.** In an AP the first term is 22, n th term is -11 and sum to first n th term is 66. Find n and d , the common difference.
- Q37.** The first term and the last term of an A.P. are -4 and 146 and the sum of the A.P. is 7171 . Find the number of terms in the A.P. and common difference.
- Q38.** Find a and d for arithmetic sequence if $S_{25} = 650$ and $a_{25} = 62$.
- Q39.** Find a and d for arithmetic sequence if $S_{31} = 5580$ and $a_{31} = 360$.
- Q40.** Find the sum of first 20 terms of an A.P. in which 3rd term is 7 and 7th term is two more than thrice of its 3rd term.
- Q41.** In an AP, the sum of first ten terms is -150 and the sum of its next ten terms is -550 . Find the AP.
- Q42.** The sum of an A.P. is 136, the common difference 4, and the last term 31 find n .
- Q43.** Find the sum of all natural numbers between 500 and 1,000 which are divisible by 13.
- Q44.** If the sum of first five terms of an AP is equal to the sum of the first ten terms of the AP. Show that the sum of its 15 terms is zero.
- Q45.** Find the sum of the first ten terms for arithmetic sequence $a_2 = 9, a_4 = 13$.
- Q46.** Find the sum $2 + 4 + 6 + \dots + 200$.
- Q47.** Find the sum $-5, -8, -11, \dots - 230$.
- Q48.** How many term of the A.P. 26, 21, 16, 11, 6, 3 are need to give the sum 11.
- Q49.** Find the sum of all 3 digits numbers which have the remainder 2, when divided by 3.
- Q50.** Find the sum of all natural numbers between 1 and 99 which are multiples of 5.
- Q51.** Solve the equation $1 + 4 + 7 + 10 + 13 + 16 + \dots + x = 287$

ADDITIONAL QUESTIONS

- Q52.** Find p, q such that 18, $p, q, 3$ are in A.P.
- Q53.** The first term of an A.P. is 5 and its 100th term is -292. Find the 50th term of this A.P.
- Q54.** Which term of the sequence 45, 41, 37, 33 is the first negative term.
- Q55.** The sum of three numbers is an A.P. is 18, and their product is 192. Find the numbers.
- Q56.** The sum of an A.P. is 136, the common difference 4, and the last term is 31, find n .
- Q57.** If the p th, q th and r th terms of an AP be a, b, c respectively then show that $a(q - r) + b(r - p) + c(p - q) = 0$
- Q58.** Show that $(a - b)^2, (a^2 + b^2)$ and $(a + b)^2$ are in AP.
- Q59.** If p th, q th and r th terms of an AP are a, b, c respectively then show that $(a - b)r + (b - c)p + (c - a)q = 0$.
- Q60.** Two AP's have the same common difference. The first term of one of these is 3 and that of other is 8. Find the difference between their 30th term.
- Q61.** Determine k so that $k^2 + 4k + 8, 2k^2 + 3k + 6, 3k^2 + 4k + 4$ are three consecutive terms of an AP
- Q62.** Prove that $a_{p+q} + a_{p-q} = 2a_p$.
- Q63.** Which term of the AP 24, 21, 18, 15, ... is the first negative term.
- Q64.** If the n th term of a progression be a linear expression in n , then prove that this progression is an A.P. [Hint : $a_n = a + b n$].
- Q65.** Find the middle term of the A.P. 10, 7, 4, -62.
- Q66.** For what value of n are the n th terms of the following two APs the same 13, 19, 25, ... and 69, 68, 67, ... ? Also find this term.
- Q67.** If the sum of the first n terms of an AP is given by $S_n = (3n^2 - n)$ find its (i) n th term (ii) first term, and (iii) common difference
- Q68.** The sum of first n terms of an AP is given by $(n^2 + 8n)$. Find the 12th term of the AP. Also find the n th term of the AP.
- Q69.** Find the number of terms of the AP 63, 60, 57, ... so that their sum is 693. Explain the double answer.
- Q70.** If the 5th and 12 terms of an AP are -4 and -18 respectively, find the sum of the first 20 terms of the AP.
- Q71.** How many term of the A.P. 26, 21, 16, 11, 6, 3 are need to give the sum 11.
- Q72.** Find the sum of all natural numbers between 250 and 1000 exactly divisible by 3.
- Q73.** The sum of first six terms of an AP is 42. The ratio of its 10th term to 30th term is 1 : 3 calculate the first and 13th term of the AP.
- Q74.** Find the sum of all two-digit odd positive integers.

CLASS X

Q75. Show that the sum of an AP whose first term is a . The second term is b and last term is equal to c is equal to $\frac{(a+c)(b+c-2a)}{2(b-c)}$.

Q76. Find the common difference of an A.P whose first term is 100 and the sum of whose first six term is five times the sum of the next six terms. (hint : $S_6 = 5(S_{12} - S_6)$).

Q77. For what value of n are the n th terms of the following two APs the same 13, 19, 25,... and 69, 68, 67,... ? Also find this term.

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