Ch – 4 Quadratic Equations

Definition :

Let P(x) be a polynomial if P(x) = 0 then it is called quadratic equation. The standard form of quadratic equation is $ax^2 + bx + c = 0$, where a,b and c are real numbers and $a \neq 0$, is called a quadratic equation in variable x

Roots of a Quadratic Equation :

Let P(x) = 0, be a Quadratic Equation, then the zeros of a Polynomial P(x) are called the roots of the Quadratic Equation.

Note :

1 : If α is a root of quadratic equation, $ax^2 + bx + c = 0$, then we say that $x = \propto$ satisfies the equation $ax^2 + bx + c = 0$ or $x = \alpha$ is a solution of the equation $ax^2 + bx + c = 0$

Derivation of Quadratic Formula :

Consider the Quadratic Equation $ax^2 + bx + c = 0$

Derivation of Formula :

Step 1 Constant term shifted towards R.H.S

$ax^2 + bx = -c$

Step 2 Whole equation divided by coeff. of x^2

$x^2 + \frac{b}{a}x = -\frac{c}{a}$; $\left(\frac{1}{2} \times \frac{b}{a} = \frac{b}{2a}\right)$

$\left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$

Step 3 Adding Both side by square of the half of the coefficient of x

$\Rightarrow x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$

$\Rightarrow x + \frac{b^2}{2a} = \frac{-4ca + b^2}{4a^2}$

Taking square root on both the side, we get

$\Rightarrow \sqrt{x + \frac{b^2}{2a}} \qquad \sqrt{\frac{b^2-4ac}{4a^2}}$

$\Rightarrow x + \frac{b}{2a} = \pm \frac{\sqrt{b^2-4ac}}{2a}$

$\Rightarrow x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$

Either ,

$\Rightarrow x = \frac{-b + \sqrt{b^2-4ac}}{2a} \Rightarrow \frac{-b + \sqrt{D}}{2a}$ or

$\Rightarrow x = \frac{-b - \sqrt{b^2-4ac}}{2a} \Rightarrow \frac{-b - \sqrt{D}}{2a}$ or

Discriminant :

If $ax^2 + bx + c = 0$, is a quadratic equation and $a \neq 0$, then the expression; $b^2 - 4ac$ is known as its discriminant and is denoted by letter D.i.e., $D = b^2 - 4ac$. where a, b and c are real numbers.

Nature of the Roots of a quadratic equation :

Nature of the roots of the quadratic equation decided by 'D'

1. If D > 0, then equation has real and unequal roots and both roots can be calculated by using the following formulae.

$x(\text{or } \alpha) = \frac{-b+\sqrt{b^2-4ac}}{2a} \Rightarrow \frac{-b+\sqrt{D}}{2a}$ or

and $x\ (\text{or } \beta) = \frac{-b-\sqrt{b^2-4ac}}{2a} \Rightarrow \frac{-b-\sqrt{D}}{2a}$ or

2. If D = 0 then equation has real and equal roots, then both roots can be calculated by using following formulae.

$x(\text{or } \alpha) = \frac{-b}{2a}$ and $x(\text{or } \beta) = \frac{-b}{2a}$

3. If D < 0 then equation has no real roots.

Important result :

On determining the value of an unknown variable in the given equation when the nature of root is given. Apply the following conditions

a. If equation has real roots then apply $D \geq 0$

b. If equation has real and different roots then apply $D > 0$.

c. If equation has real and equal roots then apply $D = 0$.

d. If equation has no real roots than apply $D < 0$

CLASS X

Practice sheet 1

Q1. Select quadratic equation from the following equation :

(i) $x^2 - 6x - 4 = 2x^2 + 2x + 7$ (ii) $7x = 2x^2 + 8x^2 + 3 + 5x$ (iii) $x^2 - 5x + 6 = 0 + 3(x-1)$

(iv) $x + \frac{7}{x} = x + 3x^{-1} + 6x + 3$ (v) $2x^2 - 5 = 0 + 3x + 12x$ (vi) $x^2 + 2x - 3 = x(x+1) + 5$

Q2. In each of the following, determine whether the given value of x is a solution of the given equation or not :

(i) $3x^2 - 2x - 1 = 0$: $x = 1$ (ii) $(2x + 3)(3x - 2) = 0$: $x = 2/3$

(iii) $2\sqrt{3}x^2 + \sqrt{3}x + 5$: $x = \sqrt{3}$ (iv) $\sqrt{7}x^2 - 6x - 13\sqrt{7}$: $x = -\sqrt{7}$

(v) $5x^2 + 33xy - 14y^2 = 0$: $x = -7y$ (vi) $2x^2 - 2\sqrt{6x} + 3 = 0$: $x = -\sqrt{3}$

Q3. In the following, find the value of k for which the given value is a solution of the given equations

(i) $x^2 - x(a + b) + k = 0$: $x = a$ (ii) $Kx^2 + \sqrt{2}x - 4 = 0$: $x = \sqrt{2}$

(iii) $2x^2 - 6kax - 2a^2 = 0$: $x = a$ (iv) $2kx^2 - 6kax + 18 = 0$: $x = -a$

(v) $ax^2 + 2\sqrt{3}\,kx - a^3 = 0$: $x = a$ (vi) $2x^2 + (6k + 1)x - 35 = 0$: $x = -2$

Q4. Find the roots of the following quadratic equations by factorization method :

(i) $2x^2 + 11x - 21 = 0$ (ii) $3x^2 - 14x + 8 = 0$

(iii) $x^2 - 2x + \frac{7}{16} = 0$ (iv) $72x^2 - \sqrt{3}x - 3 = 0$

(v) $3\sqrt{2}\,x^2 - 10x + 4\sqrt{2} = 0$ (vi) $\sqrt{3}\,x^2 + (\sqrt{3} + 1)x + 1 = 0$

(vii) $9a^2x^2 + 5abx - 4b^2 = 0$ (viii) $3a^2x^2 + 12abx + 9b^2 = 0$

(ix) $x^2 - (a + b)x + ab = 0$ (x) $x^2 - (a + b)x + b = 0$

(xi) $\frac{1}{x-a-b} + \frac{1}{a} + \frac{1}{b} = \frac{1}{x}$ (xii) $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$

(xiii) $x^2 + \left(\frac{a}{a+b} + \frac{a+b}{a}\right)x + 1 = 0$ (xiv) $p^2x^2 + (p^2 - q^2)x - q^2 = 0$

(xv) $\sqrt{x+6} - \sqrt{x+2} = 4$ (xvi) $4x + \frac{1}{x} = 25\frac{1}{25}$

(xvii) $3^{x+2} + 3^{-x} = 10$ (xviii) $\sqrt{x+3} = 2x$

(xix) $\left(\frac{x}{x+1}\right)^2 - 5\left(\frac{x}{x+1}\right) + 6 = 0$ (xx) $\frac{x}{x+1} + \frac{x+1}{x} = \frac{34}{15}$

(xxi) $\sqrt{x} - \frac{1}{\sqrt{x}} = \frac{48}{7}$ (xxii) $\frac{x+3}{x-2} - \frac{1-x}{x} = \frac{17}{4}$

(xiv) $\frac{1}{x-2} + \frac{2}{x-3} = \frac{6}{x}$ (xv) $p^4x^2 + 2p^2x - 24 = 0$

Practice sheet 2

Q5. Solve the following quadratic equation by using quadratic formula :

(i) $x^2 - 2ax + (a^2 - b^2) = 0$ (iii) $a(x^2 + 1) - x(a^2 + 1) = 0$ (v) $36x^2 - 12ax + (a^2 - b^2) = 0$

(ii) $(a^2 - b^2)x^2 + 4bx - 4 = 0$ (iv) $2a^2x^2 - 6a^2b^2x + 4b^2 = 0$

Q6. Find the value of k for each of the following quadratic equation, so that they have two equal roots or coincident roots or perfect square.

(i) $2x^2 - 10x + k = 0$ (iii) $kx^2 - 2\sqrt{5}x + 4 = 0$ (v) $x^2 + kx + 6 = 0$

(ii) $9x^2 + 3kx + 4 = 0$ (iv) $4x^2 + kx + 9 = 0$ (vi) $kx^2 - 5x + k = 0$

Q7. Find the value of K for which the following equation has real roots.

(i) $3x^2 + 2x + k = 0$ (iii) $9x^2 + 3kx + 4 = 0$ (v) $5x^2 - kx + 1 = 0$

(ii) $2x^2 + kx - 4 = 0$ (iv) $2x^2 - kx - 4 = 0$

Q8. Find the value of K for which the following equation has real and distinct roots.

(i) $kx^2 + 2x + 1 = 0$ (iii) $2x^2 - 5x - k = 0$ (v) $x^2 - kx + 9 = 0$

(ii) $4x^2 - 3kx + 1 = 0$ (iv) $2x^2 + kx + 2 = 0$

Q9. Find the value of K for which the following equation has no real roots.

(i) $5x^2 - kx + 1 = 0$ (iii) $3x^2 + 2x + k = 0$ (v) $2x^2 - 5x - k = 0$

(ii) $kx^2 - 6x - 2 = 0$ (iv) $4x^2 - 3kx + 1 = 0$

Q10. Find the value of k for which the equation $9x^2 - 24x + k = 0$ has equal roots. Also find the roots.

Q11. If -4 is a root of the quadratic equation $x^2 + px - 4$ and the quadratic equation $x^2 + px + k = 0$ has equal roots, find the value of k

Q12. Show that the equation, $x^2 + ax - 4$ has real and distinct roots for all real values of a

Q13. If -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, find the value of k.

Q14. Find the least positive value of k for which the equation $x^2 + kx + 4$ has real roots.

Q15. Find the positive value of k for which the equation $x^2 + kx + 64 = 0$ and will have real roots.

Q16. If the equation $(1 + m^2)x^2 + 2mcx + (c^2 - a^2) = 0$ has equal roots, prove that $c^2 = a^2(1 + m^2)$.

Q17. Show that the equation $2(a^2 + b^2)x^2 + 2(a + b)x + 1 = 0$ has no real roots.

Q18. If one root of the quadratic equation $2x^2 + kx - 6 = 0$ is 2 , find the value of k. Also, find the other root.

Q19. If x = 2 and x = 3 are the rots of the equation $3x^2 - 2kx + 2m = 0$ find the value of k and m.

Q20. If one roots of the equation $4x^2 - 2x + (\lambda - 4) = 0$ be the reciprocal of the other, then find the value of λ .

Practice sheet 3

Based on numbers

(i) The sum of a number and its reciprocal is $\frac{10}{3}$,find the numbers.

(ii) The sum of two numbers is 15. If the sum of their reciprocal is 3 / 10, find the two numbers.

(iii) Determine the two consecutive even positive integers the sum of whose squares is 100.

(iv) If the sum of square of three consecutive multiples of 5 is 1250. Find these numbers.

(v) Divide 16 into two parts such that twice the square of the large part exceeds the square of the smaller part by 164.

Based on Two digit numbers

(i) A number consists of two digits. Units digit is greater than tenth place digit by 8. If the number is more than the product of its digit by 10, find the number.

(ii) A two digit number is such that the product of its digit is 12. When 36 is added to this number. The digits inter change their places. Find the numbers.

(iii) A two digit number is such that the product of the digits is 16. When 54 is subtracted from the number, the digits are interchanged. Find the number.

(iv) A two digit number is 4 times the sum of its digit and twice the product of its digits. Find the number.

(v) A two digit number such that the tenth digit is 2 more than one's digit if the product of digits is 63. Find the number.

Based on Area of rectangle & triangle

(i) Vikarm, wisheds to fit three rods together in the shape of a right triangle. The hypotenuse is to be 2cm longer than the base and 4cm longer than the altitude. What should be the length of the rods.

(ii) The diagonal of a rectangular field is 60m more than the shortest side. If the longer side is 30m more than the shorter side, find the sides of the field.

(iii) A chess board contains 64 equal squares and the area of. Each square is 6.25 cm2 . A border around the board is 2cm. Wide find the length of the side of the chess board.

(iv) The area of a right angled triangle is 165m2 . Determine its base and altitude. If the latter exceeds the former by 7m.

(v) The hypotenuse of a right angled triangle is 20m. If the difference between the length of the other side be 4m, find the other sides.

Based on Cost

(i) Rs.250 were divided equally among a certain number of children. If there were 25 children more, each would have received 50 paise less. Find the number of children.

(ii) Rs.6500 were divided equally among a certain number of persons. had there been 15 more persons, each would have got Rs.30 less. Find the original number of persons.

(iii) Rs.9000 were divided equally among a certain number of persons. Had there been 20 more persons, each would have got Rs.160 less. Find the original number of persons.

(iv) A man buys a number of pens for Rs.80. If he had bought 4 more pens for the same amount, each pen would have cost him Rs.1 less. How many pens did he buy.

(v) If the price of a book is reduced by Rs.5, a person can buy 5 more books for Rs.300. Find the original list price of the book.

Based on Speed

(i) A train covers a distance of 90 km at a uniform speed. Had the speed been 15km / hr more, it would have taken half an hour less for the journey. Find the original speed of the train.

(ii) A train travels a distance of 300 km at a constant speed. If the speed of the train is increased by 5 km/hr, the journey would have taken 2 hours less. Find the original speed.

(iii) A passenger train takes 3 hours less for a journey of 360 km if its speed is increase by 10 km/hr from its usual speed. What is its usual speed.

(iv) If I had walked 1 km/hr faster, I would have taken 10 minutes less to walk 12 km. Find the rate of my walking.

(v) The speed of a boat in still water is 11 km/ hr. It can go 12km upstream and return downstream to the original point in 2 hours 45 minutes. Find the speed of stream.

Based on Age

(i) The difference of a mother's age and her daugther's age is 21years and the twelveth part of product of their ages is less than the mother's age by 18 year. Find the ages.

(ii) If a boy's age and his father's amount together its 24 years and the fourth part of the product of their ages, exceeds the boy's age by 9 years. Find how old they are.

(iii) The product of Ramu's age five years ago and his age nine years later is 15. Determine Ramu's present age.

(iv) The sum of the ages of a father and his son 54 years. Two years ago, the product of their ages was 400. Find their present ages.

(v) Is the given situation is possible if so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years is 120.

ADDITIONAL QUESTIONS

Q21. Two pipes running together can fill a cistern in $3\frac{1}{13}$ minutes if one pipe taken 3 minutes more than the other to fill. It find the time in which each pipe would fill the cistern.

Q22. A takes 10 days less than the time taken by B to finish a piece of work. If both A and B together can finish the work in 12 days. Find the time taken by B to finish work.

Q23. The angry Arjun carried some arrows for fighting with Bheeshm. With half the arrows, he cut down the arrows thrown by Bheeshm on him and with six other arrows he killed the rath driver of Bheeshm. With one arrow each he knocked down respectively the rath, flag and the bow of Bheeshm. Finally, with one more than four times the square root of arrows he laid Bheeshm unconscious on an arrow bed. Find the total number of arrows Arjun had.

Q24. One-fourth of a herd of camels was seen in the forest. Twice the square root of the herd had gone to mountains and the remaining 15 camels were seen on the bank of a river. Find the total number of camels.

Q25. If the list price of a toy is reduced by Rs.2, a person can buy 2 toys more for Rs.360. Find the original price of the toy.

Q26. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of her marks would have been 210. Find her marks in two subjects.

Q27. Solve for x : $x = \frac{1}{2-\frac{1}{2-\frac{1}{2-x}}}$

Q28. Solve for x : $\sqrt{6+\sqrt{6+\sqrt{6\ldots}}}$

Q29.Prove that the equation $x^2 (a^2 + b^2) + 2x (ac + cd) + (c^2 + d^2) = 0$ has no real roots .

Q30. If the roots of the equation $ax^2 + bx + c = 0$ and $bx^2 - 2\sqrt{acx} + b = 0$ are simultaneously real, prove that $b^2 = ac$.

Q31. Solve for x : $2\left(\frac{x+2}{2x-3}\right) - 9\left(\frac{2x-3}{x+2}\right) = 3$

Q32. For what value of k , $(4 – k)x^2 + (2k + 4)x + (8k + 1) = 0$ is a perfect square.

Q33. Find the roots of the following quadratic equation by factorization method :

$$\frac{2}{x^2} - \frac{5}{x} + 2 = 0$$

Q34. Find the value of k if the following quadratic equation has real and equal roots.

$Kx^2 + kx + 1 = 0 – 4x^2 – x$

Q35. Find the roots of the following quadratic equation by factorization method :

$x^2 + x – (a + 1) (a + 2) = 0$

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