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Concept Sheet

PAIR OF LINERS EQUATIONS IN TWO VARIABLES

1. A combination of constants and variables, connected by four fundamental arithmetical operations + − × , , and ÷, is called an algebraic expression. e.g., $6x^2 - 5y^2 + 2xy$ is an **algebraic expression.**

2. An algebraic expression with sign 'equal to'(=) , is called an **equation**. Without an equal sign, it is an expression only.

e.g., 6x + 12 = 0, is an equation but only 6x + 12 is an expression.

3. An equation which involves only one variable with highest power 1, is called a **linear equation** in that variable

e.g., Each of the equation given below is a linear equation in one variable.

(i) 5x + 18 = 3 + 4x

(ii) $\frac{y-1}{2} - \frac{y+5}{4} = 6 - y$

4. A value of the variable which when substituted for the variable in the equation, makes its two sides equal, is called a **solution** (or root) of the linear equation in one variable.

5. Any pair of values of x and y which satisfies the equation ax + by + c = 0 , $a \neq 0$, $b \neq 0$, is called a solution of linear equation of two variables.

6.Two linear equations in the same variables x and y, are called **pair of linear equations** in two variables. The general form of pair of linear equations in two variables x and y is

$$a_1x + b_1y + c_1 = 0$$

and

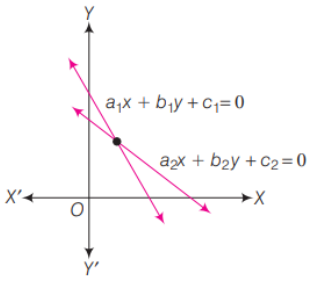
$$a_2x + b_2y + c_2 = 0$$

7. Let us consider two linear equations as

$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

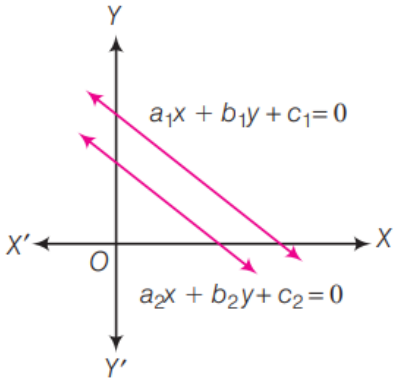
Case I Two lines will intersect at one point. If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then the graph will represent two intersecting lines. In this case, there is only one point of intersection which is **unique**. A pair of linear equations in two variables having unique solution, is called consistent pair of linear equations.

Hence, intersecting lines have one solution.



Case II Two lines will not intersect i.e., they are parallel.

If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the graph will represent two parallel lines. Thus, system has no solution. A pair of linear equations having **no solution**, is called **inconsistent pair** of linear equations.



Case III Two lines will be coincident,

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the graph will represent two coincident lines. Thus, system has **infinitely many solutions**. (Q there are infinite points of intersection) A pair of linear equations having infinite distinct solutions, is called **dependent pair** of linear equations.

8. **Behaviour of lines representing a pair of linear equations in two variables and conditions for consistency**

Pair of lines	$\frac{a_1}{a_2}$	$\frac{b_1}{b_2}$	$\frac{c_1}{c_2}$	Compare the ratios	Graphical representation	Algebraic interpretation	Consistency
$x-2y=0$ $3x+4y-20=0$	$\frac{1}{3}$	$\frac{-2}{4}$	$\frac{0}{-20}$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Exactly one solution (unique)	System is consistent
$2x+3y-9=0$ $4x+6y-18=0$	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{-9}{-18}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinitely many solutions	System is consistent (dependent)
$x+2y-4=0$ $2x+4y-12=0$	$\frac{1}{2}$	$\frac{2}{4}$	$\frac{-4}{-12}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution	System is inconsistent

9. **Substitution Method**

Steps used in this method to solve a pair of linear equations are given below.

Step I Find the value of one variable say x (or y) in terms of other variable i e. ., y (or x) form an equation.

Step II Substitute this value of x (or y) in other equation, then it reduces to a linear equation in one variable i.e., in terms of y (or x) which can be solved easily.

Step III Substitute the value of y (or x) obtained in step II in the equation which is used to obtain the value of the other variable in step I.

10**. Elimination Method**

Steps used in this method to solve a pair of linear equations are given below:

Step I Firstly, make the coefficient of one variable (x or y) numerically equal by multiplying both equations by some suitable non-zero constant.

Step II Now, add or subtract both equations, so that one variable is eliminated and remaining equation has one variable.

Step III Solve the equation in one variable to get the value of this variable (x or y).

Step IV Substitute this value (x or y) in any one of the given equations to get the value of other variable.

Practice Sheet 1

Q1. Write the condition for a pair of linear equations : $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_1 = 0$ to have a unique solution.

Q2. Write the condition for a pair of linear equations : $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_1 = 0$ to have a infinite number of solutions.

Q3. What is the condition for a system of linear equations : $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_1 = 0$ to be inconsistent ?

Q4. Write the general form of a system of two simultaneous linear equations in two variables a and y ?

Q5. What is the name given to a system of simultaneous linear equations, if it has no solution ?

Q6. What is the minimum number of solutions that a system of simultaneous linear equations must have, if it is a consistent system?

Q7. What is the maximum number of solutions that a system of simultaneous linear equations can have, if it is a consistent system?

Q8. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$ and without drawing the graph of the pair of linear equations x – 4y + 5 = 0 , 2x -3y + 5 = 0 , show that the lines represented by these equations intersect at a point.

Q9. What is the nature of graph of a system of linear equations which is consistent independent?

Q10. What is the nature of graph of a system of linear equations which is inconsistent?

Q11. The path of a train A is given by the equation 3x + 4y – 12= 0 and the path of another train B is given by the equation 6x + 8y – 48 = 0. Represent this situation graphically?

Q12. Given the linear equation 2x + 3y – 8 = 0, write another linear equation in two variables such that geometrical representation of the pair so formed is

(i) Intersecting lines
(ii) Parallel lines
(iii) Coincident lines

Practice Sheet 2

Q13. Solve the equations by drawing their graphs. Determine whether these are consistent (independent or dependent) or inconsistent: 2x + y - 4 = 0 and -2x + y + 3 = 0.

Q14. Find graphically the vertices of a triangle whose sides have following equations: 2y - x = 8, 5y - x = 14 and y - 2x = 1.

Q15. Find graphically the vertices of a triangle whose sides have the following equations: y = x, y = 0 and 2x + 3y = 10

Q16. Use a single graph paper and draw the graph of the following equations: y = x, y = -x and 2x + 3y = 6. Shade the triangle formed by these lines and find the vertices of triangle so formed.

Q17. Draw the graphs of the following equations on same graph paper: 3x - 2y = 6, 3x + y = 15. Find coordinates of the vertices of the triangle formed by the two straight lines and x axis.

Q18. Show graphically that the system of equations : 2x + 4y = 10 ; 3x + 6y = 12, has no solution.

Q19. Show graphically that the system of equations : 3x – y = 2 ; 9x – 3y = 6, has infinitely many solutions.

Q20. Use a single graph paper and draw the graph of the following equations : 2y – x = 8 ; 5y – x = 14, y – 2x = 1.

Q21. Solve the following system of equations graphically : x + 3y = 6 ; 2x – 3y = 12 and hence find the value of a, if 4x + 3y = a.

Q22. Solve the following system of linear equations graphically : 2x – y – 4 = 0 ; x + y + 1 = 0. Find the points where the lines meet y - axis.

Q23. Draw the graphs of 2x + y = 6 and 2x – y + 2 = 0. Shade the region bounded by these lines and x - axis. Find the area of the shaded region.

Practice Sheet 3

Q24. Solve each of the following system of the equations by substitution method .

(i) $2x + y - 35 = 0$
$3x + 4y - 65 = 0$

(ii) $ax + by = a - b$
$bx - ay = a + b$

(iii) $x + y = a + b$
$ax - by = a^2 - b^2$

(vii) $ax + by = a^2$
$bx - ay = b^2$

(iv) $x + 2y + 1 = 0$
$2x - 3y - 12 = 0$

(v) $2x - y = 6$
$x - y = 2$

(vi) $ax + by = a - b$
$bx - ay = a + b$

(vii) $\frac{x}{a} + \frac{y}{b} = 2$
$ax - by = a^2 - b^2$

Q25.Solve each of the following system of the equations by elimination method

(i) $2x + 3y = 17$
$3x - 2y = 6$

(ii) $2x + y - 35 = 0$
$3x + 4y - 65 = 0$

(iii) $\frac{x}{a} + \frac{y}{b} = a + b$
$\frac{x}{a^2} + \frac{y}{b^2} = 2$

(iv) $(a - b)x + (a + b)y = 2a^2 - 2b^2$
$(a + b)(x + y) = 4ab$

(v) $4x + 7y = 20$
$21x - 13y = 21$

(vi) $23x - 17y + 11 = 0$
$31x + 13y - 57 = 0$

(vii) $\frac{5x}{3} + \frac{3y}{5} - 1 = 0$
$\frac{5x}{3} - \frac{3y}{5} + 2 = 0$

(viii) $ax + by - a + b = 0$
$bx - ay - a - b = 0$

(ix) $\frac{x}{7} + \frac{y}{3} = 5$
$\frac{x}{2} - \frac{y}{9} = 6$

(x) $0.5x + 0.7y = 0.7$
$0.3x + 0.5y = 0.50$

(xi) $2x - y - 3 = 0$
$4x + y - 3 = 0$

(xii) $\frac{x}{a} + \frac{y}{b} = 2$
$ax - by = a^2 - b^2$

(xiii) $(a + 2b)x + (2a - b)y = 2$
$(a - 2b)x + (2a + b)y = 3$

(xiv) $a^2x + b^2y = c^2$
$b^2x + a^2y = d^2$

(xv) $x + ay = b$
$ax - by = c$

(xvi) $mx - ny = m^2 + n^2$
$x + y = 2m$

(xvii) $\frac{x}{a} + \frac{y}{b} = a + b$
$\frac{x}{a^2} + \frac{y}{b^2} = 2$

(xviii) $a^2x + b^2y = c^2$
$b^2x + a^2y = d^2$

(xix) $\frac{11x - 5y}{11} = \frac{3x + y}{3}$
$8x - 5y = 1$

(xx) $0.8x + 0.3y = 3.8$
$0.4x + 0.5y = 0.6$

(xxi) $x + y = 5xy$
$3x + 2y = 13xy$

(xxii) $\frac{x+y}{xy} = 2$
$\frac{x-y}{xy} = 2$

(xxiii) $\frac{x+y}{2} - \frac{x-y}{3} = 8$
$\frac{x+y}{3} + \frac{x-y}{3} = 11$

(xxiv) $\frac{2}{x} + \frac{3}{y} = 13$
$\frac{5}{x} - \frac{4}{y} = -2$

(xxv) $99x + 101y = 499$
$101x + 99y = 501$

(xxvi) $31x + 43y = 117$
$43x + 31y = 105$

(xxvii) $5ax + 6by = 28$
$3ax + 4by = 18$

Practice Sheet 4

Q26. Find the value of k for which the following system of linear equations has an infinite number of solutions :
$5x + 2y = k, 2$;
$(k + 1)x + ky = 3k + 4$

Q27. Find the value of k so that the following system of linear equations has an infinite number of solutions :
$(k + 1)x + 2ky = k + 3$;
$(4k + 1)\, x + 12y = 15$

Q28. Find the value of k so that the following system of linear equations has an infinite number of solutions :
$x + (k + 2)y = 4$;
$(2k - 1)\, x + 25y = 6k + 2$

Q29. For what values of a and b, the following system of linear equations has an infinite number of solutions.
$3x - y = 14$;
$(a + b)x - 2by = 5a + 2b + 1$

Q30. Find the values of a and b for which the following system of linear equations has an infinite number of solutions :
$2x – 3y = 7$
$(a + b)x – (a + b – 3)y = 4a + b$

Q31. For what value of k will the equation $x + 2y + 7 = 0$, $2x + ky + 14 = 0$. represent coincident lines?

Q32. For what values of m and n, the following system of linear equations has an infinite number of solutions :
$3x + 4y = 12$;
$(m + n)x + 2(m - n)y = 5m – 1$

Q33. For what value of k, will the system of equations
$x + 2y = 5$;
$3x + ky – 15 = 0$
has (i) unique solution (ii) no solution

Q34. Determine the values of k so that the following linear equations have no solution :
$(3k + 1)\, x + 3y – 2 = 0$
$(k^2 + 1)x + (k – 2)\, y – 5 = 0$

Q35. For what value of k will the linear equations has no solution ?
$3x + y = 1$; $(2k – 1)x + (k – 1)y = 2k + 1$

Q36. Find the value of k for which each of the following system infinite
$kx + 3y = 2k + 1$; $2(k + 1)\, x + 9y = 7k + 1$

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Practice Sheet 5

Word problems based on cost

(i) 2 audio cassettes and 3 video cassettes cost Rs.340 and 3 audio cassettes and 2 video cassettes cost Rs. 260. What are the price of an audio cassette and of a video cassette.

(ii) From Delhi station if we buy 2 tickets to station A and 3 tickets to station B, the total cost is Rs.77, but if we buy 3 tickets to station A and 5 tickets to station B, the total cost is Rs124. What are the fares from Delhi to station A and to station B.

(iii) A and B each have certain number of oranges. A says to B "if you give me 10 of your oranges left with you, B replies. If you give me 10 of your oranges. I will have twice the number of oranges, I will have the same number of oranges as left with you" find the number of oranges with A and B separately.

(iv) 2 chairs and 2 tables cost Rs700 and 5 chairs and 3 tables cost Rs.1100. What is the total cost of one chair and one table?

(v) The sum of the prices of an almirah and a table is Rs2340 and the difference of their prices is Rs 140. Find the price of each.

(vi) A bag contains 94 coins of 50-P and 25-P denominations. If the total worth of these coins be Rs. 29.75, find the number of coins of each kind.

(vii) A leading library has a fixed charge for the first three days and an additional charge for each day thereafter. Shruti paid Rs.27 for a book kept for seven days, while Susy paid Rs.21 for the book she kept for 5 days. Find the fixed charge and the charge for each extra day.

(viii) The Taxy charges in a city comprise for a fixed charge together with the charge for the distance covered for a journey of 10 km the charge paid is Rs.75 and for a journey of 15 km the charge paid is Rs.110. What will be person have to pay for travelling a distance of 25 km.

(ix) A railway half - ticket costs half the full fare but the reservation charges are the same on a half - tickets as on a full tickets. One reserved first class ticket from station A to B costs Rs.2125. Also, one reserved first class ticket and one reserved half first class ticket from A to B costs Rs.3200. Find the full fare station A to B and also the reservation charges for a ticket.

(x) A part of monthly hostel charges in a college are fixed and the remaining depend on the number of days one has taken food in the mess. When a student A takes food for 20 days, he has to pay Rs.1000 as hostal charges whereas a student B, who takes food for 26 days, pays Rs.1180 as hostel charges. Find the fixed charge and the cost of food per day.

Word problems based on Age

(i) Five years ago, A was thrice as old as B and 10 years later A shall be twice as old as B. What are the present ages of A and B

(ii) One year ago a man was four times as old as his son. After 6 years, his age exceeds twice his son's age by 9 years. Find their present ages.

(iii) I am three times as old as my son. Five years later I shall be two and-a half times as old as my son. How old am I and how old my son.

(iv) A man has 5 sons, the sum of the ages of the 5 sons being equal to the age of the father. In 12 years time the sum of ages of the sons will be double that of their father. What is the father's age now.

(v) The age of the father is 3 years more than three times the age of the son. Three years hence, father's age will be 10 years more than twice the age of the son. Determine their present ages.

(vi) 10 years ago the age of a father was four times that of his son. 5 years ago the age of father was three times that of the sun. Find the present ages of father and the son.

(vii) A year ago, the father was 8 times as old as his son. Now his age is the square of his son's age. Find their present ages.

(viii) The sum of the present ages of father and his daughter is 60 years and difference of twice the ages of father before 5 years and the age of daughter after 5 years is 60 years. Find their present ages.

(ix) Three times the age of my father added to seven times the age of my sister is 183 years. Six times the difference between their ages added to 9 years is three times the sum of their ages. Determine their present age.

(x) If twice the son's age in years is added to the age of his father, the sum is 90. if twice the Father's age in years is added to the age of the son. The sum is 120. Find their ages.

Word problems based on Fractions

(i) A fraction becomes 4/5 if 1 is added to both numerator and denominator. If however 5 is subtracted from both numerator and denominator, the fraction becomes 1/2. What is the fraction.

(ii) If we add 1 to the numerator and subtract 1 from the denominator a fraction becomes 1. It also becomes 1/2 if we only add 1 to the denominator. What is the fraction.

(iii) A fraction is such that if the numerator is multiplied by 2 and the denominator is increased by 2, we get 5/4 but if the numerator is increased by 1 and the denominator is doubled, we get 1/2. Find the fraction.

(iv) A fraction is such that if the numerator is multiplied by 3 and the denominator is reduced by 3, we get 8/2 ; but if the numerator is increased by 8 and the denominator is doubled, we get 2/5 , Find the fraction.

(v) If the numerator of the fraction is multiplied by 2 and its denominator is increased by 2, it becomes 6/7 if instead we multiply the denominator by 2 and increases the numerator by 2 it reduces to 1/2 what is fraction.

(vi) The denominator of a fraction is 4 more than twice the numerator. When both the numerator an denominator are decreased by 6, then the denominator become 12 times the numerator, determine the fraction.

(vii) The sum of the numerator and denominator of a fraction is 4 more than twice the numerator. If the numerator and denominator are increased by 3, they are in the ratio 2 : 3. Determine the fraction.

(viii) The sum of the numerator and denominator of a fraction is 3 less than twice the denominator. If the numerator and denominator are decreased by 1, the numerator becomes half the denominator. Determine the fraction.

(ix) The sum of the numerator and denominator of a fraction is 12. If the denominator is increased by 3, the fraction becomes 1/2 . Find the fraction.

(x) If 2 is added to the numerator of a fraction, it reduces to 1/2 and if 1 is subtracted from the denominator, it reduces to 1/3 . Find the fraction.

Word problems based on Two digit numbers

(i) Seven times a two-digit number is equal to four times the number obtained by reversing the order of digits. If the difference between the digits is 3, find the number.

(ii) The sum of a two digit number and the number obtained by reversing the order of its digits is 121, and two digits differ by 3. Find the number.

(iii) The sum of the digits of a two digit number is 8. The number obtained by interchanging the two digits exceeds the given number by 36. Find the number.

(iv) A two-digit number is obtained by either multiplying sum of the digits by 8 and adding 1 or by multiplying the difference of the digits by 13 and adding 2. Find the no.

(v) The sum of a two - digit number and the number obtained by interchanging the digits of the number is 110. The digits of the number differ by 6. How many such numbers are there ? Find them.

(vi) The sum of the digits of a two digit number is 9. The number obtained by reversing the order of digits of the given number exceeds the given number by 27. Find the given number.

(vii) The sum of two numbers is 8. If their sum is four times their difference, find the numbers.

(viii) If 2 is added to each of two given numbers, their ratio becomes 1 : 2. However, if 4 is subtracted from each of the given numbers, the ratio becomes 5 : 11. Find the numbers.

(ix) The sum of a two digit number and the number formed by interchanging its digits is 110. If 10 is subtracted from the first number, the new number is 4 more than 5 times the sum of the digits in the first number. Find the first number.

(x) A two digit number is 4 times the sum of its digits and twice the product of the digits. Find the number.

Word problems based on Quadrilateral & triangle

(i) The largest angle of a triangle is twice the sum of the other two, the smallest is one-forth of the largest. Find the angles in degrees.

(ii) In a ΔABC, $\angle A = x^o$, $\angle B = (3x - 2)^o$, $\angle C = y^o$ .Also $\angle C - \angle B = 9^o$ .Find the three angles.

(iii) In a cyclic quadrilateral ABCD $\angle A = (2x + 4)^o$, $\angle B = (y + 3)^o$, $\angle C = (2y + 10)^o$, $\angle D = (4x - 5)^o$, find the four angles.

(iv) The area of a rectangle gets reduced 9 square units, if its length is reduced by 5 units and the breath is increased by 3 units. If we increase the length by 3 units and the breath by 2 units, then the area is increased by 67 square units. Find the length and the breath of the rectangle.

(v) In a rectangle the length is increased and the breath is reduced by 2 units each, the area is reduced by 28 square units, if the length is reduced by 1 units, and breath increased by 2 units, the area increases by 33 square units. Find the dimensions of the rectangle.

(vi) In a triangle, the sum of the two angles is equal to the third, if the difference between them is 50, determine the angles.

(vii) An obtuse angle of a parallelogram is twice its acute angle. Find the measure of each angle of the parallelogram.

(viii) Half the perimeter of a garden, whose length is 4 more than its width is 36m. Find the dimensions of the garden.

(ix) In ΔABC, $\angle A = x^0$, $\angle B = 3x^0$ and $\angle C = y^0$ .If $3y - 5x = 30$, prove that the triangle is right angles.

(x) Find the four angles of a cyclic quadrilateral ABCD n which $\angle A = (x + y + 10)^0$, $\angle C = (y + 20)^0$, $\angle A = (x + y - 50)^0$ and $\angle A = (x + y)^0$.

(ADDITIONAL QUESTIONS)

(i) A boat goes 30km upstream and 44km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstream. Find the speed of the stream and that of the boat in still water.

(ii) Anuj travels 600 km to his home, partly by train and partly by car. He takes 8 hours when he travels 120 km by train and the rest by car. He takes 20 minutes longer if he travels 200 km by train and rest by car. Find the speed of the train and the car.

(iii) A boat goes 12 km upstream and 40 km down stream in 8 hours. It can go 16 km upstream and 32 km down stream in the same times, find the speed of the boat in still water and the speed of the stream.

(iv) A man walks a certain distance at a certain rate. Had he walked 1/2 km per hour faster, he would have taken 1 hours less. But, if he had gone 1 km an hour slower, he would have taken 3 hours longer. Find the distance covered by the man and his original rate of walking.

(v) A person can row 8km upstream and 24 km down stream in 4 hours. He can row 12 km down stream and 12 km upstream in 4 hours. Find the speed of the person in still water as also the speed of the current.

(vi) Points A and B are 90 km apart from each other on a highway. A car starts from A and another from B at the same time. If they go in the same direction they meet in 9 hours and if they go in opposite direction they meet in 9 / 7 hours. Find their speeds.

(vii) Places A and B are 80 km apart from each other on a highway. A car starts from A and other from B at the same time. If they more in the same direction, they meet in 8 hours and if they move in opposite direction, they meet in 1 hours and 20 minutes. Find the speeds of the cars.

(viii) Places P_1 and P_2 are 250km apart from each other on a national highway. A car starts from P_1 and another from P_2 at the same time. If they go in the same direction they they meet in 5 hours and if they go in opposite directions they meet in 25 / 13 hours. Find their speeds.

(ix) Solve each of the following system of the equations by equating the coefficient method.

$(a - b)x + (a + b)y = a^2 - 2ab - b^2$
$(a + b)(x + y) = a^2 + b^2$

(x) For what values of a and b, the following system of linear equations has an infinite number of solutions :

$2x + 3y = 7;$
$(a + b + 1)x + (a + 2b + 2)y = 4(a + b) + 1.$

(xi) Solve the following system of equations graphically : $x + 3y = 6$; $2x – 3y = 12$ and hence find the value of a, if $4x + 3y = a$. Find the points where the lines meet y – axis .

(xii) Draw the graphs of $2x + y = 6$ and $2x – y + 2 = 0$. Shade the region bounded by these lines and x - axis. Find the area of the shaded region.

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