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# Concept Sheet

## Polynomials

1. A polynomial in one variable x, is an algebraic expression of the form

$P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots\ldots + a_2 x^2 + a_1 x + a_0$

where, $a_0, a_1, a_2 \ldots\ldots, a_n$ are constant and these are also called **coefficients** of polynomial and $a_i \neq 0$, where i = a,1,…n .

The exponent of the highest degree term in a polynomial, is known as its **degree**.

e.g., $f(x) = 5x + \frac{1}{3}$ is a polynomial in variable x of degree 1.

2. (i) A polynomial of degree zero, is called **zero polynomial.**

Or

A polynomial which contains only constant term, is called a **zero polynomial**.

**Note** The degree of zero polynomial is not defined.

(ii) A polynomial of degree one, is called **linear polynomial.**

(iii) A polynomial of degree two, is called **quadratic polynomial.** Every quadratic polynomial can have atmost two zeroes.

(iv) A polynomial of degree three, is called **cubic polynomial**.

(v) A polynomial of degree four, is called **biquadratic polynomial**.

3. A real number k is said to be a zero of a polynomial f (x), if f (k) = 0.

e.g. $\frac{-7}{5}$ is a zero of polynomial p(x) = 5x + 7 , since $p\left(\frac{-7}{5}\right) = 0$ .

4. (i) A linear polynomial has atmost one zero.

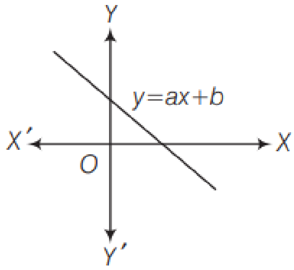
(ii) A quadratic polynomial has atmost two zeroes. In general, a polynomial of degree n has atmost n zeroes.

(iii) If a polynomial p (x) of degree n, has n zeroes than its graph will intersect X-axis at n different points.

(iv) Zero of a polynomial p (x) is precisely the abscissa of the point where the graph intersects X-axis.

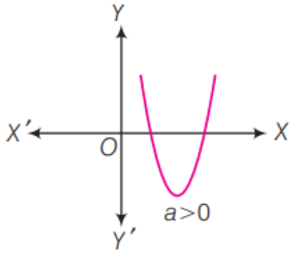
5. **Graph of a Linear Polynomial** Let us consider a linear polynomial y = ax + b .

All these values of x and y are marked on the graph paper. By joining all the markings we get the required graph, which is found to be a straight line.

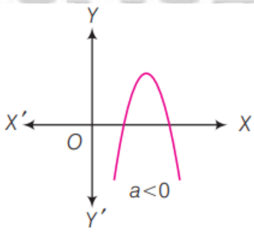



6. **Graph of a Quadratic Polynomial** The graph of a quadratic polynomial ( $ax^2 + bx + c$ ) is ∪-shaped, called as parabola.

(i) If $a > 0$ in polynomial $ax^2 + bx + c$ , then the shape of parabola is **opening upward** (∪).

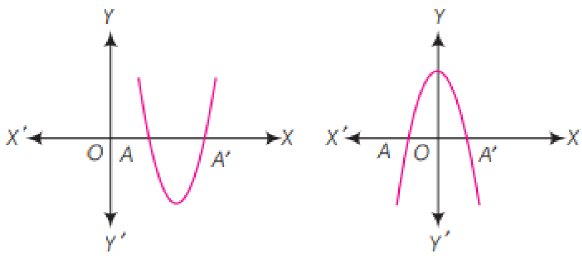



(ii) If $a < 0$ in polynomial $ax^2 + bx + c$ , then the shape of parabola is **opening downward (∩).**

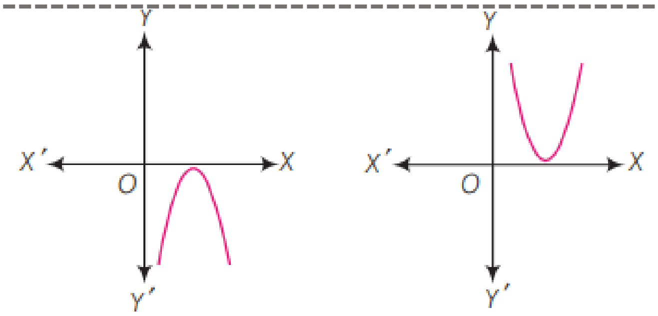



7. The graph of axhas three cases, which are given below.

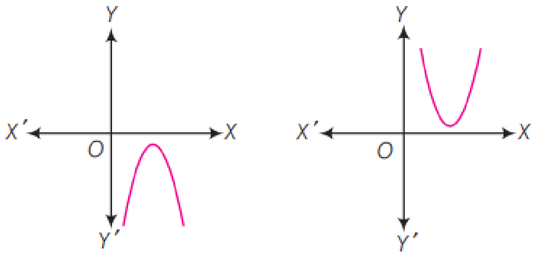
**Case I** If graph of $f(x) = ax^2 + bx + c$ intersect the X-axis at two distinct points A and A', then x-coordinate of points of intersection of curves are known as **'zeroes'** of f (x).




**Case II** If graph of $f(x) = ax^2 + bx + c$ touch the X-axis at one point only, then f (x)will have only one 'zero' the x-coordinate of point of contact.



**Case III** If graph of f(x) = $ax^2 + bx + c$ neither touch nor intersect the X-axis, then f (x)will not have any real zeroes.



## 8. Relationship between the Zeroes and the Coefficients of a Polynomial .

| Types of polynomial | General form | Number of zeroes | Relationship between zeroes and coefficients |
|---|---|---|---|
| Linear | $ax + b; a \neq 0$ | 1 | $x = -\frac{b}{a}$, i.e., $x = -\frac{\text{(Constant term)}}{\text{(Coefficient of } x)}$ |
| Quadratic | $ax^2 + bx + c; a \neq 0$ | 2 (say $\alpha, \beta$) | Sum of zeroes $(\alpha + \beta) = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{b}{a}$ |
| | | | Product of zeroes $(\alpha\beta) = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{c}{a}$ |
| Cubic | $ax^3 + bx^2 + cx + d; a \neq 0$ | 3(say $\alpha$, $\beta$ and $\gamma$) | Sum of zeroes $(\alpha + \beta + \gamma) = -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3} = -\frac{b}{a}$ |
| | | | Sum of product of zeroes taken two at a time $(\alpha\beta + \beta\gamma + \gamma\alpha) = +\frac{\text{Coefficient of } x}{\text{Coefficient of } x^3} = \frac{c}{a}$ Product of zeroes $(\alpha\beta\gamma) = -\frac{\text{Constant term}}{\text{Coefficient of } x^3} = -\frac{d}{a}$ |

9. (i) If α and β are the zeroes of a quadratic polynomial, then quadratic polynomial will be k [$x^2$ – (sum of zeroes)x + product of zeroes]

i.e., $k[x^2 - (\alpha + \beta)x + \alpha\beta], k \neq 0$

(ii) If α, β and γ are the zeroes of a cubic polynomial, then cubic polynomial will be

k[ $x^3$ - (sum of zeroes) $x^2$ + (sum of product of zeroes two at a time) x – product of zeroes]

i.e.., $k[x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma]$,

where k is a non - zero real number .

## 10. FORMULA lIST

| | |
|---|---|
| (i) $( a + b )^2 = a^2 + b^2 + 2ab$ | (vii) $( a - b )^3 = a^3 - b^3 - 3ab( a - b )$<br>$= a^3 - b^3 - 3a^2b + 3ab^2$ |
| (ii) $( a - b )^2 = a^2 - b^2 + 2ab$ | (viii) $( a^3 + b^3 ) = ( a + b ) ( a^2 + b^2 – ab )$ |
| (iii) $( a + b )^2 - ( a – b )^2 = 4ab$<br>And $( a + b )^2 + ( a – b )^2 = 2 ( a^2 + b^2 )$ | (ix ) $( a^3 – b^3 ) = (a – b ) ( a^2 + b^2 + ab )$ |
| (iv) $(a^2 – b^2 ) = ( a + b ) ( a – b )$ | (x) $( a + b + c )^2 = a^2 + b^2 + c^2 + 2( ab + bc + ac )$ |
| (v) $( a + b )^3 = a^3 + b^3 + 3ab( a + b )$<br>$= a^3 + b^3 + 3a^2b + 3ab^2$ | (x) $[ a^3 + b^3 + c^3 - 3abc ] = ( a + b + c )$<br>$( a^2 + b^2 + c^2 – ab – bc – ac)$ |
| (vi) If $a + b + c = 0$ , then $a^3 + b^3 + c^3 = 3abc$ | |

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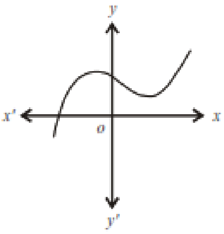
https://chat.whatsapp.com/HTcfeKqE4wN8075HOehy0t

# Practice Sheet 1

**Q1**. Look at the graphs below for some polynomials for each graph, find number of zeros of p(x) .

(i)

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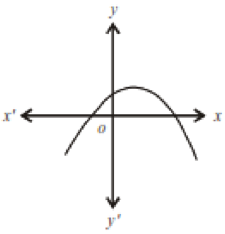
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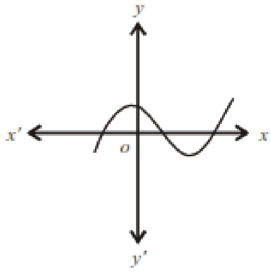
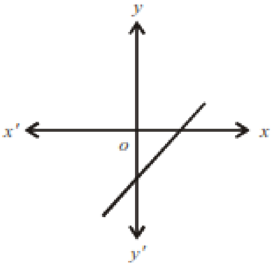
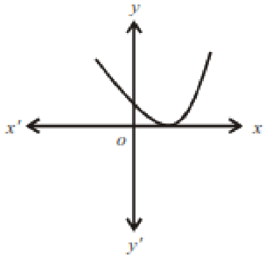
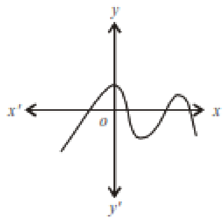

(iv)

(v)

(vi)


**Q2.**Find the zeroes of the polynomial and verify the relationship between the zeroes and their coefficient.

(i) $x^2 - 2x - 8$
(ii) $4s^2 - 4s + 1$
(iii) $x^2 - 3x + 2$
(iv) $6x^2 - 7x - 3$
(v) $3x^2 - 14x + 8$
(vi) $4x^2 - 4x - 3$

**Q3**.Find the zeros of the following polynomials and verify the relationship between zeros and its coefficient.

(i) $6x^2 - 15$
(ii) $3x^2 - 2$
(iii) $32x^2 - 8$
(iv) $36x^2 - 49$
(v) $49x^2 - 16$
(vi) ) $8x^2 - 3$

**Q4**.Find the zeros of the following polynomials and verify the relationship between zeros and its coefficient.

(i) $9b^2 x^2 - 16a^2$
(ii) $x^2 - 16a^2$
(iii) $16a^2x^2 - 25b^2$
(iv) $4a^2x^2 - 9b^2$
(v) $0.25x^2 - 0.16b^2$
(vi) $25a^4b^4 x^2 - 16p^2$
(vii) $16p^2 q^2 x^2 - 121z^2$
(viii) $0.01a^2x^2 - 0.25z^2$
(ix) $0.04x^2 - 0.16$

**Q5**.Find the zeros of the following polynomials and verify the relationship between zeros and its coefficient.

(i) $a^2x^2 + 2abx + b^2$
(ii) $4x^2 + 4ax + a^2$
(iii) $6a^2x^2 + 16ab^2 + ab^2$
(iv) $12a^2b^2x^2 + 10abx - 2$
(v) $9a^4 x^2 + 5a^2 x - 4$
(vi) $4a^2x^2 - 4a^2b^2 x - 8b^4$

**Q6**.Find the zeros of the following polynomials and verify the relationship between zeros and its coefficient.

(i) $3\sqrt{7}x^2 - 10x + \sqrt{7}$
(ii) $\sqrt{13}x^2 - 10x - 3\sqrt{13}$
(iii) $6\sqrt{7}x^2 - x - \sqrt{7}$
(iv) $5\sqrt{3}x^2 - x - 6\sqrt{3}$
(v) $4x^2 + 4\sqrt{2}x - 6$
(vi) $9x^2 - 8\sqrt{3}x + 4$

# Practice sheet 2

**Q7**. Is 2 is a zero of the polynomial $p(x) = x^2 – 5x + 6$ .

**Q8**. For what value of p, (–4) is a zero of the polynomial $p(x) = x^2 – 2x – ( 7p + 3)$ .

**Q9**. If one zero of the quadratic polynomial $2x^2 + px + 4$ is 2, find the other zeros. Also find the value of p.

**Q10**. Find the quadratic polynomial whose zeros are $\frac{2}{3}$ and $\frac{-1}{4}$ . Verify the relation between the coefficients and the zeros of the polynomial.

**Q11.** Form a quadratic polynomial whose zeroes are $3 + \sqrt{5}$ and $3 - \sqrt{5}$ .

**Q12**. Form a quadratic polynomial whose zeroes are $\frac{3-\sqrt{3}}{5}$ and $\frac{3+\sqrt{3}}{5}$ .

**Q13**. Form a quadratic polynomial whose zeroes are $-2\sqrt{3}$ and $-\sqrt{3}$.

**Q14**. Form a quadratic polynomial whose one zeroes is $\sqrt{5}$ and the product of the zeroes is $-2\sqrt{5}$ .

**Q15**. Find a quadratic polynomial, the sum of whose zeroes is $\frac{5}{2}$ and their product is 1. Hence find the zeros of the polynomial.

**Q16**. Find a quadratic polynomial. The sum of whose zeros is –1 and their product is –1.

**Q17**. Form a quadratic polynomial whose zeros are 5 and 6.

**Q18**. If $\alpha$ and $\beta$ are the zeros of the polynomial $p(x) = 2x^2 + 6x – 8$ . Form a quadratic polynomial whose zeros are $-\alpha$ and $-\beta$

**Q19**. If $\alpha$ and $\beta$ are the zeros of polynomials $p(x) = 6x^2 – 18x + 12$ . Form a quadratic polynomial whose zeros are $-5\alpha$ and $-5\beta$.

**Q.20**. If $\alpha$ and $\beta$ are the zeros of the polynomial $P(x) = 6x^2 + 18x – 36$ . Form a quadratic polynomial whose zeros are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$.

**Q21.**If $\alpha$ and $\beta$ are the zeros of the polynomial $P(x) = 7x^2 – 5x – 8$ . Form a quadratic polynomial whose zeros are $(\alpha +1)$ and $(\beta+1)$ .

**Q22**. If $\alpha$ and $\beta$ are the zeros are the polynomial $P(x) = 6x^2 – 7x + 8$ . Form a quadratic polynomial whose zeros are $(\alpha + 2)$ and $(\beta + 2)$.

**Q23**. If $\alpha$ & $\beta$ are the zeros of the polynomial $P(x) = 2x^2 – 3x + 5$ . Form a quadratic polynomial whose zeros are $(2\alpha + 2\beta)$ and $(\alpha +\beta)$.

**Q24**. If $\alpha$ and $\beta$ are the zeros of the polynomial $p(x) = 3x^2 – 4x + 9$ . Form a quadratic polynomial whose zeros are $(\alpha\beta +1)$ and $(\alpha\beta -1)$.

**Q25**.Verify that $\frac{1}{2}$ , $-\frac{1}{3}$ and $\frac{2}{5}$ are zeros of cubic polynomial(x) $=30x^3 – 17x^2 – 3x + 2$ . Also verify the relationship between the zeros and their coefficient.

**Q26**. Verify that $\sqrt{3}$,- $\sqrt{3}$ and 4 are zeros of cubic polynomial $p(x) = x^3 – 4x^2 – 3x + 12$ . Also verify the relationship between the zeros and their coefficient.

**Q27**. If $\alpha$ and $\beta$ are the zeros of the polynomial of $P(x)= 2x^2 – 5x + 7$ . Find the value of following expression

(i) $\alpha\beta$ (ii) $\alpha + \beta$ (iii) $\alpha^2\beta^2$

(iv) $\alpha^{-3}\beta^{-3}$ (v) $\alpha - \beta$ (vi) $\alpha^2 + \beta^2$

(vii) $\alpha^3 + \beta^3$ (viii) $\alpha^{-1} + \beta^{-1}$ (ix) $\alpha^{-2}\beta^{-2}$

(x) $\alpha^{-2} + \beta^{-2}$ (xi) $\alpha^2\beta + \alpha\beta^2$ (xii) $\alpha^3\beta + \beta^3\alpha$

(xiii) $\frac{1}{\alpha} + \frac{1}{\beta}$ (xiv) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ (xv) $\frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2}$

(xvi) $\frac{\alpha^{-2}}{\beta^{-1}} + \frac{\beta^{-2}}{\alpha^{-1}}$ (xvii) $\frac{1}{\alpha^{-3}} + \frac{1}{\beta^{-3}}$ (xviii) $\frac{\alpha^3}{\beta} + \frac{\beta^3}{\alpha}$

(xix) $\alpha^{-4} + \beta^{-4}$ (xx) $\frac{\alpha^{-5}}{\beta^{-3}} + \frac{\beta^{-5}}{\alpha^{-3}}$ (xxi) $\frac{\alpha^{-7} \times \beta^{-7}}{\alpha^2\beta^2}$

# Practice sheet 3(Additional Questions)

**Q.28** Find the value of p , for which one root of the quadratic polynomial $px^2 - 14x + 8 = 0$ is 6 times the other.

**Q29.**If one root of the quadratic equation $3x^2 + px + 4 = 0$ is $\frac{2}{3}$, then value of p and the other root of the equation.

**Q30.**Find the value of k such that the polynomial $x^2 - (k + 6)x + 2(2k - 1)$ has sum of its zeroes equal to half of their product.

**Q31**. The roots $\alpha$ and $\beta$ of the quadratic equation $x^2 - 5x + 3(k - 1) = 0$ are such that $\alpha - \beta = 1$ .Find the value of k.

**Q32**.If the zeroes of the polynomial $x^2 + px + q$ are double in value to the zeroes of the polynomial $2x^2 - 5x - 3$ , then find the value of p and q .

**Q33**.Polynomial $x^4 + 7x^3 + 7x^2 + px + q$ is exactly divisible by $x^2 + 7x + 12$ , then find the value of p and q .

**Q34.** If $\alpha$ and $\beta$ are the zeroes of the polynomial $p(x) = 2x^2 + 5x + k$ satisfying the relation $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$, then find the value of k .

**Q35**.If x-1 is a factor of the polynomial $p(x) = x^3 + ax^2 + 2b$ and $a + b = 4$ , then find a and b.

**Q36**.If $\alpha$ , $\beta$ are the zeroes of the quadratic polynomial $p(x) = x^2 - (k + 6)x + 2(2k - 1)$ , then find the value of k , if $\alpha + \beta = \frac{1}{2}\alpha\beta$ .

**Q37**. P and q re the zeroes of the polynomial $4y^2 - 4y + 1$ . What is the value of $\frac{1}{p} + \frac{1}{q} + pq$ ?

**Q38.**If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2 , then find the value of k .

**Q39.**If $\alpha$ *and* $\beta$ are the zeroes of the polynomial $f(x) = px^2 - 2x + 3p$ and $\alpha$ *and* $\beta = \alpha\beta$ , then find p .

**Q40**. If $\alpha$ and $\beta$ are the zeroes of the quadratic polynomial $P(x) = Kx^2 + 4x + 4$ such that $\alpha^2 + \beta^2 = 24$, find the value of K.

**Q41**. If sum of the squares of zeroes of the quadratic polynomial $p(x) = x^2 - 8x + K$ is 40, find the value of K.

**Q42**. If one root of the polynomial $p(x) = 5x^2 + 13x + K$ is reciprocal of the other. Find the value of K.