CHAPTER 1

Number System

We, humans, speak in a particular language. This language is made up of words and letters. Although, we type words and letters in the computer, the computer does not understand the words and letters.

When we type a character (letters, digits or special characters) on the keyboard, the computer converts and stores it in the form of strings of 0s and 1s. The symbols 0 and 1 are together called binary digits or bits, and form the binary number system. The memory of a computer can be thought of as a vast group of cells. Each of these cells contains one bit of information, i.e. each cell contains 0 or 1.

We will Learn

- Understanding Number System
- Conversion to Decimal Number System
- Conversion of Decimal Numbers to Other Number Systems
- Computer Arithmetic

Understanding Number System

A number system is a set of values used to represent different quantities such as number of students in a class or number of viewers watching a particular show, etc.

We, humans, use decimal number system in our day-to-day life whereas, a computer represents all kinds of data and information in binary number system. The total number of digits used in a number system is called its Base or Radix.

Some important number systems are as follows :

Decimal Number System : The decimal number system is what humans use most commonly. It is composed of 10 numerals – 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. Using these digits we can express any quantity. It is also called the base 10 system because it uses 10 digits.

The decimal system is a positional value system in which the value of a digit depends on its position. The rightmost digit has the least positional value (weight), therefore, it is called the least significant digit (LSD). The leftmost digit has the maximum positional value (weight), therefore, it is called the most significant digit (MSD).

For example, the number 256 can be represented in the following way

Positional Values (weights)	10^2	10^1	10^0
	2	5	6
	MSD		LSD

$$256 = 2 \times 10^2 + 5 \times 10^1 + 6 \times 10^0$$
$$= 2 \times 100 + 5 \times 10 + 6 \times 1$$
$$= 200 + 50 + 6$$

Binary Number System : The computer system is designed on the binary number system. The binary number system has a base of 2. It has only two digits, 0 and 1, and can represent any character with these two digits. The binary number system is also a positional number system wherein each binary digit has its own value or weight, which is expressed as a power of 2.

More To Know

Gottfried Leibniz, a German Mathematician is credited with the invention of the modern Binary number system.

Consider a fractional binary number, 1011.0101.

Positional Values (weights)	2^3	2^2	2^1	2^0		2^{-1}	2^{-2}	2^{-3}	2^{-4}	
MSD	1	0	1	1	.	0	1	0	1	LSD

For a binary number, the binary point separates the whole number part from the fractional part. The place values of the digits to the right of the binary point are obtained by raising 2 to successive negative powers.

Octal Number System : The octal number system has a base of 8. It has eight digits 0, 1, 2, 3, 4, 5, 6 and 7. Each digit of an octal number can have any value from 0 to 7. The octal number system is also a positional numbering system. Each octal digit has its own positional value or weight, which is expressed as the power of 8.

Consider 1267.12, an octal number with a fraction.

Positional Values (weights)	8^3	8^2	8^1	8^0		8^{-1}	8^{-2}	
MSD	1	2	6	7	.	1	2	LSD

The place value of each digit to the left and the right of the radix point is equal to 8 raised to the successive positive or negative powers, respectively.

Hexadecimal Number System : This number system consists of 16 digits, numbers 0–9 and the letters A–F, where A–F represent decimal numbers from 10 to 15. The base of this number system is 16. This number system is also known as Hex. It is also a positional numbering system. The value of a hexadecimal digit is expressed as the power of 16.

Consider the hexadecimal fractional number, A65.C2. The positional value of each digit is a power of 16.

Positional Values (weights)	16^2	16^1	16^0		16^{-1}	16^{-2}	
MSD	A	6	5	.	C	2	LSD

Conversion to Decimal Number System

Numbers in the binary, octal or hexadecimal number systems can be converted to decimal numbers. The steps to do this conversion are :

1. Find the positional value of each digit.
2. Multiply each digit with its positional value, starting from the extreme right digit. Increase the power one by one, keeping the base fixed.
3. Sum up all products calculated in step 2.
4. The total is the equivalent value in the decimal number system.

Binary number to decimal number conversion

A binary number can be converted to its equivalent decimal number by adding up the product of each digit value (0 or 1) and its positional value, as shown below.

Example : $(111001)_2 = (1 \times 2^5) + (1 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$

$= 32 + 16 + 8 + 0 + 0 + 1$

$= (57)_{10}$

Hence, the decimal equivalent of $(111001)_2$ is $(57)_{10}$.

Octal number to decimal number conversion

To change any octal number into decimal number, we have to start multiplying the digits of the number from right hand side with increasing powers of 8 starting from 0 and finally summing up the products.

Example : $(123)_8 = (1 \times 8^2) + (2 \times 8^1) + (3 \times 8^0)$

$= 64 + 16 + 3$

$= (83)_{10}$

Hence, the decimal equivalent of $(123)_8$ is $(83)_{10}$.

Hexadecimal number to decimal number conversion

Since we are using a base-16 number system, start multiplying the digits of the number from right hand side with increasing powers of 16 starting from 0 and finally summing up the products.

Example : $(C19)_{16} = (C \times 16^2) + (1 \times 16^1) + (9 \times 16^0)$

$= (12 \times 16^2) + (1 \times 16^1) + (9 \times 16^0)$ [substitute C with 12]

$= 12 \times 256 + 16 + 9 = (3097)_{10}$

Hence, the decimal equivalent of $(C19)_{16}$ is $(3097)_{10}$.

Conversion of Decimal Numbers to Other Number Systems

To convert a decimal number into other numbers, follow the given rules :

1. Divide the given decimal number with the required base.
2. Write down the remainder and divide the quotient again with the same base.
3. Repeat the step 2 till the quotient is zero.
4. Write the remainders obtained in each step in the reverse order to form the required number system.

Decimal number to binary number conversion

Let us convert 345 in binary notation. Note, that the desired base is 2, so we have to repeatedly divide the given decimal number by 2.

		Remainder
2	345	
2	172	→ 1
2	86	→ 0
2	43	→ 0
2	21	→ 1
2	10	→ 1
2	5	→ 0
2	2	→ 1
2	1	→ 0
	0	→ 1

Hence, the binary equivalent of $(345)_{10}$ is $(101011001)_2$.

Decimal number to octal number conversion

Now, express the same decimal number 345 in octal notation.

		Remainder
8	345	
8	43	→ 1
8	5	→ 3
	0	→ 5

Hence, the octal equivalent of $(345)_{10}$ is $(531)_8$.

Decimal number to hexadecimal number conversion

This time express the same decimal number 345 in hexadecimal notation.

		Remainder
16	345	
16	21	→ 9
16	1	→ 5
	0	→ 1

Hence, the hexadecimal equivalent of $(345)_{10} = (159)_{16}$.

Remember, if the remainder is greater than 9, replace it with appropriate letter A, B, C, D, E, F accordingly.

Example : Convert the decimal number 1341 to hexadecimal notation.

		Remainder
16	1341	
16	83	→ 13
16	5	→ 3
	0	→ 5

Replace 13 with 'D', while writing the answer.

Hence, $(1341)_{10} = (53D)_{16}$.

Computer Arithmetic

As a computer understands only the binary code, the data input to the computer by the user is converted into binary code for processing. This processing may involve various kinds of arithmetic operations, such as addition, subtraction, multiplication, division, etc on the binary numbers.

Binary Addition : The technique used to add binary numbers inside the computer is very easy and simple. This is performed in the same way as we perform addition with decimal numbers.

The following table illustrates the addition of two binary digits :

a	b	a + b = c
0	0	0 + 0 = 0
0	1	0 + 1 = 1
1	0	1 + 0 = 1
1	1	1 + 1 = 0 with 1 carry

Example : **Add :** $(1000)_2 + (1111)_2$

```
   1000
+  1111
  10111
```

Add : $(11111)_2 + (1011)_2$

```
    1 1 1 1
    1 1 1 1 1
  + 0 1 0 1 1
   10 1 0 1 0
```

Binary Subtraction : The rules given in the table must be followed to perform binary subtraction.

a	b	a – b = c
0	0	0 – 0 = 0
0	1	0 – 1 = 1 (with borrow)
1	0	1 – 0 = 1
1	1	1 – 1 = 0

Example : **Subtract :** $(1010)_2$ from $(1111)_2$

```
   1111
 – 1010
   0101
```

Subtract : $(11)_2$ from $(1100)_2$

```
     0 1 10
   1 1 0 0
 – 0 0 1 1
   1 0 0 1
```

Looking Back

- Information in computer is stored in the form of strings of 0s and 1s.
- The most commonly used number systems in digital representation are the decimal, binary, octal and hexadecimal.
- The decimal number system is composed of 10 numerals – 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.
- The binary number system has a base of 2 with only two digits, 0 and 1.
- The octal number system has the base 8.
- The hexadecimal number system has the base 16.
- In the decimal, binary, octal and hexadecimal systems, the value of any digit in a number depends on its position within that number, i.e. the place value of the digit.

EVALUATE

A. Multiple Choice Questions

1. The total number of digits used in a number system is called its ____________ .
   (a) base ✓ (b) bass (c) bace
2. A computer understands only ____________ digits.
   (a) binary ✓ (b) decimal (c) octal
3. To convert decimal number into binary number, divide the number by :
   (a) 2 ✓ (b) 8 (c) 10
4. In binary addition, 1+ 0 equals to :
   (a) 0 (b) 1 ✓ (c) 10
5. Which of these is not a hexadecimal number?
   (a) ABGH ✓ (b) CD23 (c) EFCD

B. Fill in the blanks.

1. The base of Decimal number system is 10.
2. Octal number system consists of 8 digits.
3. In binary Subtraction, 1 – 0 equals to 1.
4. Hexadecimal no. system uses 16 symbols to represent numbers.
5. The base of binary number system is 2 .

C. State True or False.

1. The decimal number system consists of 10 digits, i.e. 0 to 9. True
2. The numbers used in Octal number system are 1 to 7. false
3. A binary number can have only two digits, 0 and 1. True
4. 1 added to 1 equals to 1. false
5. A computer can understand human language. false

D. Convert the following.

1. $(68)_{10} = (1000100)_2$
2. $(657)_{10} = (1221)_8$ $(1010010001)_2$
3. $(10101)_2 = (21)_{10}$
4. $(4D2)_{16} = (1234)_{10}$

E. Answer the following questions.

1. Explain number system and its types.

CLASS - 9th - CHAPTER - (7) - NUMBER SYSTEM

NUMBER SYSTEM :- It is a set of values that is used to represent quantities.

Types of Number System :-

(i) Binary Number System.

→ It contain only two digit 0 and 1.

→ Base is 2.

(ii) Decimal Number System.

→ It contain only 10 digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 only.

→ Base is 10.

(iii) Octal Number System.

→ It contain only 8 digits 0, 1, 2, 3, 4, 5, 6, 7.

→ Base is 8.

(iv) Hexadecimal Number System.

→ It contain 16 digits, Number 0-9 and letter 10-A, 11-B, 12-C, 13-D, 14-E, 15-F.

→ Base is 16.

DECIMAL TO BINARY CONVERSION

(i) $(68)_{10} = (\quad)_2$

2	68	0
2	34	0
2	17	1
2	8	0
2	4	0
2	2	0
	1	

upward

$= (1000100)_2$

(ii) $(657)_{10} = (\quad)_8$

2	657	1
2	328	0
2	164	0
2	82	0
2	41	1
2	20	0
2	10	0
2	5	1
2	2	0
	1	

upward

$= (1010010001)_2$

(iii) $(345)_{10}$

2	345	1
2	172	0
2	86	0
2	43	1
2	21	1
2	10	0
2	5	1
2	2	0
	1	

upward

$= (101011001)_2$

BINARY TO DECIMAL CONVERSION

(i) $(111001)_2$

Number of terms = 6

$$= 1\times2^{n-1} + 1\times2^{n-2} + 1\times2^{n-3} + 0\times2^{n-4} + 0\times2^{n-5} + 1\times2^{n-6}$$

$$= 1\times2^{6-1} + 1\times2^{6-2} + 1\times2^{6-3} \cancel{1\times2^{6-3}} + 0\times2^{6-4} + 0\times2^{6-5} + 1\times2^{6-6}$$

$$= 1\times2^{5} + 1\times2^{4} + 1\times2^{3} + 0\times2^{2} + 0\times2^{1} + 1\times2^{0}$$

$$= 1\times32 + 1\times16 + 1\times8 + 0\times4 + 0\times2 + 1\times1$$

$(\because a^0 = 1)$

$$= 32 + 16 + 8 + 0 + 0 + 1$$

$$= (57)_{10}$$

(ii) $(10101)_2$

number of terms (n) = 5

$$= 1\times2^{n-1} + 0\times2^{n-2} + 1\times2^{n-3} + 0\times2^{n-4} + 1\times2^{n-5}$$

$$= 1\times2^{5-1} + 0\times2^{5-2} + 1\times2^{5-3} + 0\times2^{5-4} + 1\times2^{5-5}$$

$$= 1\times2^{4} + 0\times2^{3} + 1\times2^{2} + 0\times2^{1} + 1\times2^{0}$$

$$= 1\times16 + 0\times8 + 1\times4 + 0\times2 + 1\times1$$

$$= 16 + 0 + 4 + 0 + 1$$

$$= (21)_{10}$$

BINARY TO OCTAL CONVERSION

$(10011011)_2 = (\quad)_8$

binary digit = 10011011

→ make triple of binary digit.

$\underbrace{10}\underbrace{011}\underbrace{011}$

010 011 011

$$010 = 0 + 1 + 0$$
$$\downarrow \quad \downarrow \quad \downarrow$$
$$2^2 + 2^1 + 2^0$$
$$\downarrow \quad \downarrow \quad \downarrow$$
$$4 \quad 2 \quad 1$$
$$(0\times4) + (1\times2) + (0\times1)$$
$$0 + 2 + 0 = 2$$

$$011 = 0 + 1 + 1$$
$$\downarrow \quad \downarrow \quad \downarrow$$
$$2^2 + 2^1 + 2^0$$
$$\downarrow \quad \downarrow \quad \downarrow$$
$$4 \quad 2 \quad 1$$
$$(0\times4) + (1\times2) + (1\times1)$$
$$0 + 2 + 1 = 3$$

we get, $(10011011)_2 = (233)_8$

BINARY TO HEXADECIMAL NUMBER

$(010010100000)_2 = (\quad)_{16}$

we know that binary value of numbers are :-

0 = 0000	6 = 0110	12(C) = 1100
1 = 0001	7 = 0111	13(D) = 1101
2 = 0010	8 = 1000	14(E) = 1110
3 = 0011	9 = 1001	15(F) = 1111
4 = 0100	(10)A = 1010	
5 = 0101	(11)B = 1011	

Steps :- (i) make set of 4 binary number.

$$\begin{array}{ccc} 0100 & 1010 & 0000 \\ \downarrow & \downarrow & \downarrow \\ 4 & A & 0 \end{array}$$

(ii) now we get $(4A0)_{16}$.

(iii) if we dont know value of Hexadecimal symbol. then calculate individualy.

(iv) $0100 = 0\times2^{4-1} + 1\times2^{4-2} + 0\times2^{4-3} + 0\times2^{4-4}$

$= 0\times2^3 + 1\times2^2 + 0\times2^1 + 0\times2^0$

$= 0 + 4 + 0 + 0$

$= 4$

DECIMAL TO OCTAL CONVERSION

$(345)_{10} = (\quad)_8$

8	345	1
8	43	3
	5	

$= (531)_8$

DECIMAL TO HEXADECIMAL CONVERSION

$(345)_{10} = (\quad)_{16}$

16	345	9
16	21	5
	1	

$= (159)_{16}$

OCTAL TO DECIMAL CONVERSION

$(123)_8 = (1 \times 8^{n-1}) + (2 \times 8^{n-2}) + (3 \times 8^{n-3})$

number of terms $(n) = 3$

$= (1 \times 8^{3-1}) + (2 \times 8^{3-2}) + (3 \times 8^{3-3})$

$= (1 \times 8^2) + (2 \times 8^1) + (3 \times 8^0)$

$= (1 \times 64) + (2 \times 8) + (3 \times 1)$

$= 64 + 16 + 3 = (83)_{10}$

HEXADECIMAL TO DECIMAL NUMBER CONVERSION

$(C19)_{16} = (C \times 16^{n-1}) + (1 \times 16^{n-2}) + (9 \times 16^{n-3})$

number of terms $(n) = 3$

$= C \times 16^{3-1} + 1 \times 16^{3-2} + 9 \times 16^{3-3}$

$= C \times 16^{2} + 1 \times 16^{1} + 9 \times 16^{0}$

$= 12 \times 256 + 1 \times 16 + 9 \times 1$

$= 3072 + 16 + 9$

$= (3097)_{10}$

BINARY ADDITION

0 + 0 = 0

0 + 1 = 1

1 + 0 = 1

1 + 1 = 0 , carry 1

Example :-

```
  1000        11111      1001101
+ 1111      + 01011   + 1000101101
------      -------   ------------
 10111       101010     1001111010
```

BINARY SUBTRACTION

0 - 0 = 0

0 - 1 = 1 (1 borrow)

1 - 0 = 1

1 - 1 = 0

Example :-

(1)

$$\begin{array}{r} 1\ 1\ 1\ 1 \\ -\ 1\ 0\ 1\ 0 \\ \hline \\ \hline \end{array}$$

(2)

$$\begin{array}{r} 1\ 1\ 0\ 0 \\ -\ 0\ 0\ 1\ 1 \\ \hline 1\ 0\ 0\ 1 \\ \hline \end{array}$$

(3)

$$\begin{array}{r} 1\ 0\ 0\ 1\ 1 \\ -\ 0\ 1\ 0\ 1\ 0 \\ \hline 0\ 1\ 0\ 0\ 1 \\ \hline \end{array}$$